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PROCEEDINGS OF THE SOCIETY OF ARTS.

THIRTY-FIFTH YEAR, 1896–97.

THURSDAY, January 14, 1897.

THE 493d meeting of the SOCIETY OF ARTS was held this day at the Institute, Mr. Blodgett, Chairman of the Executive Committee, presiding.

The Chairman announced that President Francis Amasa Walker had died at his home early in the morning of January 5. The Secretary read a letter from the Executive Committee of the Alumni Association of the Institute, inviting the SOCIETY OF ARTS to coöperate with the Alumni Association in a memorial service. It was voted to refer this communication to the Executive Committee, with power to act as may seem best. On motion of Colonel Hewins, it was voted that a committee of three be appointed by the Chair to draft resolutions in regard to the death of President Walker, and to report at a subsequent meeting. The Chair appointed the following committee: Colonel E. H. Hewins, Professor E. C. Pickering, and Mr. Thomas Doane.

The Society then proceeded to the election of new members.

Messrs. John D. Hobart, of Malden, Albert P. Norris, of Cambridge, W. Lyman Underwood, of Belmont, and David B. Weston, of Watertown, were duly elected Associate Members.

There being no further business, the Chair introduced Mr. Samuel Cabot, of the Corporation of the Institute, who read a paper on "The

Study of the Air for Locomotive Purposes." He pointed out that the study of the soaring of birds forms the basis for the study of the problem of flight by man. The difficulty of the problem is greatly increased when it is attempted to construct machines large enough for practical purposes, the difficulties of starting and stopping being especially great. The importance of momentum in flight was dwelt upon at some length. A large Malay kite and a number of small models of flying machines were exhibited, and the speaker closed with a description of a device of his own invention for applying power to flying machines, in which two pairs of wings are caused to revolve in opposite directions upon a single shaft. A discussion followed, and then after a vote of thanks the Society adjourned.

THURSDAY, February 11, 1897.

The 494th meeting of the SOCIETY OF ARTS was held at the Institute this day at 8 P.M., Mr. Blodgett in the chair.

The record of the previous meeting was read and approved.

Colonel Hewins, Chairman of the committee appointed to draft resolutions in regard to the death of President Walker, presented the following report,[1] which was adopted and ordered to be incorporated in the records:

In accordance with the inscrutable purposes of the Ruler of the Universe, Francis Amasa Walker has been removed by death while yet in the full vigor of life and in the midst of cherished labors only partially completed and in some respects only foreshadowed. In this we realize the frailty of human life and endeavor. The SOCIETY OF ARTS has lost, not only its learned and accomplished President, but a man whose vigor and earnestness have greatly promoted its influence. Whether as soldier, student of social and political economy, as educator, or organizer, we may look to the example set by General Walker for encouragement and instruction in order to increase our usefulness in the world. So may we strive to emulate him according to our abilities and opportunities. Not only has this Society lost its valued head, but the loss to the community, while felt, cannot be wholly measured or known. In so far as we may, we would join in the bereavement of the afflicted family, to whom it must be a material consolation to remember his far-reaching labors, and that his life has been a benefit to mankind.

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[1] Resolutions adopted by the Corporation and the Faculty follow, p. 206.

There being no further business, the Chairman introduced Mr. Alvan G. Clark, of Cambridge, who read a paper on "Telescopes, with Special Reference to the Making of the Yerkes Telescope for the University of Chicago." The history of telescope making was outlined, and it was shown how the size of lenses has been limited by imperfections in the art of glass making. The methods employed in grinding the 40-inch lens for the Yerkes telescope were then described, special stress being laid upon the methods of testing and finishing the glass, for this is the most important part of the work, and requires truly artistic skill. The paper was illustrated by an exhibition of apparatus and specimens of glass. The thanks of the Society were voted to the speaker, and then the meeting adjourned.

THURSDAY, February 25, 1897.

The 495th meeting of the SOCIETY OF ARTS was held this day at the Institute at 8 P.M., Mr. Blodgett in the chair. The record of the previous meeting was read and approved.

Messrs. George W. Rolfe and Alexander W. Moseley were duly elected Associate Members. The Chairman called attention to the "Life and Letters of William Barton Rogers," recently published by Mrs. Rogers, and announced that she had given to the Institute a copy for each of the libraries.

Professor William H. Niles read a paper on "The Evolution of the Great Lakes and St. Lawrence Waterway." Attention was called to the multitude of smaller lakes, which, associated with the larger ones, constitute a system of lakes rather than of streams. It was noticed that the features of the streams as well as the number and arrangement of the lakes show that the system still retains many features of youthfulness. It was not until the close of the mediæval or mesozoic era of the earth's history that the hydrographical basin was closed on the west, and that the first boundaries of the antecedent lake system were established. During the tertiary period the humidity of the climate supplied an abundance of water, while the elevation of the lands gave velocity to the outward flowing streams. The geographic changes resulted in giving coolness to the climate, which with its humidity favored the formation of glaciers. When the ice sheet began to recede from its southern limit, it left large moraines which served

to obstruct the valleys and to cause the waters to accumulate between them and the retreating ice front. It was with these glacial lakes that the present system was initiated. In the early stages the water covered considerable areas of what are now dry land bordering on the lake system. The outlet of these waters was first in the region of Chicago towards the Mississippi River. Later, when the ice had retreated somewhat and the land to the eastward had subsided, an outlet was formed along the valley of the Mohawk to the Hudson. Still later the waters flowed southward from Oswego into this channel of the Mohawk, and at a still later time water flowed through the valley of Lake Champlain southward to the Hudson valley. It was shown that these ancient river channels now afford favorable locations for large canals, and opportunities for the development of waterways between the Great Lakes and the Atlantic seaboard of the United States.

It was voted to extend the speaker the thanks of the Society, and the meeting adjourned.

THURSDAY, March 11, 1897.

The 496th meeting of the SOCIETY OF ARTS was held at the Institute this day at 8 P.M., Professor Cross in the chair. The record of the previous meeting was read and approved.

Dr. Francis H. Williams read a paper on "Some Uses of the Röntgen Rays," in which he gave an outline of the results of his work upon the application of these rays to medical diagnosis. He pointed out that the permeability of a substance to the rays depends upon its thickness and upon its chemical composition; the permeability being less, generally speaking, the higher the atomic weight. The average for the soft tissues is not far from that of water. Calculi can be detected by means of the rays only when they are composed largely of inorganic material. The difference in permeability of the organs and tissues produces the contrast which makes them visible. The method of taking radiographs and the process of examining patients by means of the fluoroscope were described and illustrated. Dr. Williams, continuing, gave an account of the method of locating a bullet, and then he described the series of observations used in making a diagnosis of diseases of the chest. It was made clear that a thorough study of normal subjects by this means is necessary

before it can be used successfully to determine pathological conditions. The paper was illustrated by experiments and lantern views. A discussion followed, after which the Society adjourned.

THURSDAY, March 25, 1897.

The 497th meeting of the SOCIETY OF ARTS was held at the Institute this day at 8 P.M., with Mr. Blodgett in the chair. The record of the previous meeting was read and approved. Messrs. C. A. Hicks, of Needham, Dexter Brackett, of Boston, N. C. Grover, of Orono, Maine, William Jackson, of Boston, A. G. Safford, of Lowell, Frederic P. Stearns, of Dorchester, Charles H. Swan, of Boston, S. E. Tinkham, of Boston, and F. I. Winslow, of Boston, were duly elected Associate Members.

The Chairman introduced Professor C. P. Brooks, Director of the Lowell Textile School, who read a paper on "Technical Education in Europe." He divided technical schools into five classes, and pointed out that, of these, the purely trade school has, perhaps, reached its highest development in Germany, where it has had a most remarkable influence in promoting the prosperity of the country. The schools of this class in Germany, France, Belgium, and Switzerland were described. This was followed by an account of the technical high schools in Germany, and of the technical universities of England, which have a similar function. The speaker passed then to a discussion of technical education in England, of which the Combination Schools of Science, Art, and Technology form the most prominent feature.

The paper was followed by a discussion, after which, on motion of Dr. Thorp, it was voted to extend the thanks of the Society to the speaker for his very interesting and instructive paper. The Society then adjourned

ROBERT P. BIGELOW, *Secretary.*

MINUTES ADOPTED BY THE CORPORATION AND BY THE FACULTY ON THE DEATH OF PRESIDENT FRANCIS A. WALKER.

THE members of the Corporation of the MASSACHUSETTS INSTITUTE OF TECHNOLOGY desire to place on their records an expression of their sorrow at the death of their late President, and their sense of the loss of one who had contributed so largely to the growth and success of the institution with which he was immediately connected, and to the cause of scientific training and education throughout the whole country.

General Francis A. Walker was elected President of the Institute May 25, 1881, at an important period in its history. Barely sixteen years had elapsed since President William B. Rogers opened the School of Industrial Science in a few rooms on Summer Street.

The plans of President Rogers were original, the result of long study and reflection, and formed a new departure in the higher education and the application of science to industry.

The process of development had gone on under the guiding hand of the great founder as rapidly as could have been expected, and was giving promise of success when age and physical infirmity compelled him to resign his place.

President Rogers, with a full knowledge of the kind of man needed for the development of the plans he had formulated, selected General Walker as his successor. He was in the prime and vigor of manhood. He had been thoroughly trained as a thinker and administrator, and had shown his capacity in positions of high importance. He fully comprehended the scheme, new and far-reaching as it was, and he believed in its possibilities. He had energy and a force of will that no obstacles could resist, and an enthusiasm which no discouragement could chill, and in November, 1881, he entered upon the discharge of his duties.

He had rare powers of discrimination in the selection of men, as shown by the character of those he called to the different departments of the institution, and he imparted to them of his own zeal and enthusiasm.

His labors were untiring, and we are the glad witnesses of the success which crowned his efforts.

During the fifteen years of General Walker's administration the number of the buildings for the use of the School of Industrial Sci-

ence has increased from one to five; the number of professors, teachers, and lecturers from thirty-seven to one hundred and fifty-six, and of pupils from two hundred and fifty-three to upwards of twelve hundred.

These facts are the outward and visible evidence of what General Walker did for the institution. His greater and real influence is not to be so measured, and can hardly be described in words. The character of the men who have graduated from the school, and their work in every part of the country from the Atlantic to the Pacific, constitute the highest tribute to the worth of our late President.

Combined with all his great qualities as educator, administrator, publicist and writer, General Walker was a man of the kindest, warmest heart and tenderest feelings. He won the affection of his Faculty and every student as well. All regarded the President as their friend. His office and his house were alike open to them, where all were welcome, and each one knew where to look for sympathy and encouragement when needing counsel or help.

The members of the Corporation will ever cherish the memory of President Walker, and they tender to his family their warmest sympathy in this their hour of bereavement and affliction.

Francis A. Walker assumed the presidency of the Massachusetts Institute of Technology in November, 1881, and died in office January 5, 1897. For more than fifteen years he lived and labored without stint or favor for its highest welfare. Suddenly his singularly varied intellectual and public career is ended.

His life touched the public service at many points. As citizens we mourn, and as members of the Faculty of the Institute over which he presided, we desire to express and record our sense of loss in the breaking of ties which years of his wisdom, sympathy, coöperation, and good will have formed and bound together.

Among the many qualities which he possessed in eminent degree we wish to record, in particular:

His leadership.

His remarkable and unusual appreciation of departments of knowledge outside his own special and personal domain.

His delicate and keen perception of the proper relations of the various courses of instruction assembled under his charge.

His discriminating and forceful interpretation of the needs and thought of the larger world, gained by a wide range of public service.

His unfailing courtesy, his kind and conciliatory spirit without sacrifice of candor or frank expression of strong conviction.

His generous recognition of the independent judgment and action

of the instructing staff in the conduct and development of their individual work.

His powerful influence over students for good, by an unconscious appeal direct to the qualities of manliness and honest endeavor, which he illustrated by personal example, winning a devoted loyalty and confidence sacredly cherished by him.

In President Walker the Institute was honored wherever he was known, so wide was the recognition of his energy, his administrative ability, and his contributions to history, education, statistics, and political economy. No man need covet a nobler monument than the record of his life.

A TRIBUTE TO THE MEMORY OF BREVET BRIGADIER GENERAL FRANCIS AMASA WALKER.[1]

By COLONEL THOMAS L. LIVERMORE.

Companion Francis Amasa Walker, Commander of this Commandery in 1883–4, died January 4, 1897, at the age of fifty-six. At the outbreak of the war, a student of law in the office of Charles Devens, Jr., in Worcester, he followed the latter into the service and enlisted under him as a private in the 15th Massachusetts Volunteers. He was appointed Sergeant Major of the regiment in August, and his brilliant character and liberal attainments brought him promotion to Captain and Assistant Adjutant General of Volunteers in September, 1861. Taking the field with General Couch, commanding the 1st Division, 4th Corps (afterwards 3d Division, 6th Corps), he made the Peninsular and Antietam campaigns on the staff of that general, and was promoted to Major and Assistant Adjutant General, August 11, 1862. When General Couch, in October, 1862, was assigned to the command of the 2d Army Corps, Major Walker passed to that corps and was identified with it from that time until he resigned from the Army in January, 1865. He was regularly assigned to be Assistant Adjutant General of the corps with the rank of Lieutenant Colonel, January 1, 1863.

The adjutant of a corps commander is his ear and voice. It is he who collects, collates and compares the statistics of numbers from day to day, and detects the increase or diminution of the fighting strength of the corps, intercepts and digests the countless communications which ascend from twenty thousand men to their commander, conducts all correspondence and frames all orders. Even in the saddle, under the enemy's fire, he must, with nerves under control and patience unruffled, catch the spirit of commands from a general, sometimes, perhaps, inflamed with the ardor of combat or oppressed with

[1] Adopted at a meeting of the Massachusetts Commandery of the Military Order of the Loyal Legion of the United States, Boston, February 3, 1897.

the weight of disaster, and translate them in clear, courteous and orderly phrase on the instant, for transmission to subordinate commanders, and withal he must, in time of need, ride the field and penetrate the battle like the youngest aide-de-camp. All these things this volunteer of twenty-two did as if he had been trained to the duties all his life. To serve as he did in this intimate capacity with Generals Couch, Warren and Hancock, soldiers schooled in the severe traditions of the regular army, who maintained the loftiest view of the dignity and methods of their profession, was not an easy duty, but Colonel Walker not only did this, through all the arduous campaigns of 1862, 1863 and 1864, excepting the few months when he was detained from the field by wounds or in the enemy's prisons, but he also won high and repeated commendation from these commanders, both for his work in the bureau and for his valor on the field of battle. He had entered the war with a New England character and training which led him to devote himself to his duty with the utmost conscientiousness, whether at the desk or in the saddle, and he gave to it a scrupulous attention to detail, an exact observance of formalities and an unremitting labor which almost excluded relaxation. Of kind heart, with winning smile and unfailing politeness, he displayed the best qualities of roundhead and cavalier together.

The intellectual powers which afterwards made him famous in another career, guided by the motives indicated in the conduct which has been outlined, overcame all the want of military education, and all the disadvantages of youth, and he began to win high commendation with his first campaign. His name received honorable mention in the reports of many battles, including Williamsburg, Fair Oaks, Malvern Hill, Fredericksburg, Chancellorsville, Bristol Station, Wilderness, Spottsylvania, Petersburg and Ream's Station. At Chancellorsville he was wounded, and at Fair Oaks General Couch reported that he "made a daring personal reconnaissance and had his horse shot under him." General Warren being temporarily assigned to the command of the 2d Army Corps, during the absence of General Hancock, found himself thrown into the intricate and rapid manœuvres between Bull Run and the Rapidan, which took place in the latter part of 1863, with a strange command and a strange staff, a position in which as corps commander he had every opportunity to appreciate the merits and to discover the defects of the adjutant general. Praise was not easily won from General Warren, but in his report of the campaign

he singled out Colonel Walker, to say of him: "Thoroughly acquainted with his office duties, so important to the operations of an army corps, he is equally willing and gallant on the field."

Colonel Walker was by General Hancock's side through the unfortunate action at Ream's Station, August 25, 1864, in his heroic efforts to retrieve the disaster which had fallen on a corps whose colors had never before suffered shame, and received honorable mention for his conduct in General Hancock's report. He there suffered the crowning misfortune of capture. General Hancock said in his report that Colonel Walker "was sent to the front with an order just before the troops were withdrawn, and, owing to the darkness, rode into the enemy's lines and was captured." To submit to the hard fate which had befallen him was intolerable to him, and, with the unfaltering resolution which characterized his whole military career, he seized the chance to escape on the march to Richmond, and painfully made his way by night to the Appomattox River, guiding his steps by the stars. Exhausted as he was, he plunged into the stream and swam until he had made the last stroke for which he had strength, only to be again taken by the enemy as he reached the shore. The hardships of captivity accomplished what campaign and battle had failed to do, and the gallant soldier and zealous patriot was compelled by the disability incurred in prison to relinquish his commission after his long career of activity, usefulness and honor. In recognition of his distinguished services and good conduct in the campaign of 1864 he was made Colonel by brevet, to date from August 1, 1864, and for gallant conduct at Chancellorsville and meritorious services during the war he was made Brigadier General by brevet, to date from March 13, 1865.

After the war General Walker devoted himself to the advancement of science and the work of education with the same zeal which characterized his military career. Under the National Government he became Chief of the Bureau of Statistics in 1869, Superintendent of the Census in 1870 and 1880, and Indian Commissioner in 1871. He became Professor of Political Economy and History in the Sheffield Scientific School of Yale College in 1872, and in 1881 he was elected President of the Massachusetts Institute of Technology, which position he filled during the rest of his life. Under him this institution grew from a small school of local reputation to a great one of international fame, and while its numbers increased three-fold, the standard and scope of its work were greatly enlarged. The enthusiasm which

President Walker imparted to the students was like that which the successful military leader inspires in his soldiers, and the cheerfulness with which these students, under his extraordinary influence, submitted themselves to the severest and most unremitting intellectual labor was a phenomenon in the history of education. While conducting the work of his office President Walker did a great amount in other fields. His writings in his favorite science of political economy created a high reputation for him in Europe as well as in America, and he was repeatedly honored with degrees by foreign universities. His lectures and addresses were many, and his services on boards and commissions for worthy public objects, and in associations for the advancement of science and the elevation of mankind, were constant. Unmindful of his personal fortunes, he was prodigal of his voice and pen in all good works where his aid was sought, and as he approached his prime the volume of his labors grew to be a torrent, and, with his fine physique and glowing imagination, he threw himself into his labors as careless of the consequences to himself as is the young soldier in the hour of victory.

Amid all the interests which made their demands upon him, his enthusiasm for the old war times was ever ready to blaze forth; his affectionate regard for his comrades in the field was unfailingly awakened with the mention of any name familiar on the rolls of his old command. He gloried in his army, in his corps, in the name of volunteer. He knew no discrimination between his comrades excepting that of merit. It was characteristic of him, in his speech when he received his honorary degree in Dublin University, to praise in enthusiastic terms the valor of his old companions in arms of the Irish Brigade, regardless of the prejudice which made many of his audience hostile listeners. The warmth of his affection for the associations of the war was manifested in his moving oration before the Society of the Army of the Potomac, in 1890, and in his eloquent eulogies of Sheridan and Devens. His brilliant history of the 2d Army Corps, written in hours seized amid a thousand duties, will always stand as a labor of love, dedicated to the fame of his fellow soldiers.

His genial and kindly presence in our commandery is fresh in our memories. In sad sincerity we can say, we shall not look upon his like again.

SCIENTIFIC WORK OF THE BOSTON PARTY ON THE SIXTH PEARY EXPEDITION TO GREENLAND.

Report B. Glacial Observations in the Umanak District, Greenland.

By GEORGE H. BARTON, S.B.

Read November 12, 1896.

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THE western coast of Greenland presents the aspect of a plateau approaching clear to the coast line with no intervening lowlands, with an average elevation of not less than two thousand feet, and with very many summits rising from four thousand to occasionally over six thousand feet altitude.

Backward from the general sea line at a distance of ten to sixty miles lies the frontal edge of the vast ice-cap, which through the explorations of Peary and Nansen is known to cover the whole of Greenland except the narrow marginal area along the coast. Very numerous deep fiords intersect this marginal strip, in some places cutting it completely into islands, as in the vicinity of Godthaab in Southern Greenland, and in the southern portion of Melville Bay in Northern Greenland. Many of the fiords penetrate to the edge of the ice-cap from which great streams of ice pour down, filling the fiords for miles with glaciers which give birth to the numerous icebergs with which the Northwest Atlantic is dotted. These glaciers present bold fronts to the water, often several miles in width and a few hundred feet in height above the surface of the water. In all cases the fiords present means of easy access to the margin of the inland ice.

MAP SHOWING THE REGIONS ABOUT UMANAK FIORDS.

NUGSUAK PENINSULA IS THE LARGER BODY OF LAND SHOWN. THE LARGER ISLAND EAST OF UMANAK IS STORÖEN. SERMIARSUT IS ON NUGSUAK OPPOSITE UMANAK. EKALUIT IS OPPOSITE IKERASAK. AGPAT ISLAND IS NORTHWEST OF SATUT. EKINGA IS WEST OF ITIVDLIARSUK GLACIERS. THE DOTTED LINES INDICATE THE ROUTES OF THE BOAT JOURNEYS AND THE JOURNEY UPON THE INLAND ICE.

Many of the larger islands and peninsulas between the fiords have their own small ice-caps which are usually detached from the main mass of the inland ice. The larger portion of the surface of the islands and peninsulas where not covered by ice is very rough and broken, being traversed by numerous small streams flowing into the fiords, which have cut sharp, deep gulches and valleys, thus rendering travel across country very difficult.

Umanak fiord is the southern of two large arms of the largest indentation of the coast, except possibly Disko Bay, a body of water lying between Swarten Huk peninsula on the north and the great Nugsuak peninsula on the south, and which is nearly bisected by the parallel of 71°. Ubekyendt and Upernivik islands separate the two arms. The highest summits on the former island reach 3,640 feet, and on the latter, 6,650. The average elevation of Nugsuak is above 3,000 feet on its northeastern side with summits rising to above 4,000 in many cases, and in one to 6,250 feet.

Umanak fiord broadens inward and then is divided into a series of smaller fiords, the more important of which are the Kangerdluarsuk, Ignerit, Itivdliarsuk, Sermilik, and Karajak fiords. Between these are islands and peninsulas with peaks rising to 3,000 and 6,400 feet. The Karajak fiord lies directly along the northeastern side of the Nugsuak peninsula, and is bounded on the northeast by the island of Umanak, which has one single sharp peak rising to 3,720 feet, the remainder of the island having an elevation of only a few hundred feet; Storoen (the great island), whose highest peak is 4,450 feet, the island of Ikerasak, low but with one isolated peak 2,550 feet, and a peninsula, not named, with elevations of 3,000 feet. All these islands and peninsulas present precipitous faces to the water often nearly vertical, to a height of 1,000 and 2,000 feet. Each of the fiords mentioned above extends back to the margin of the inland ice, and their upper portions are occupied by glaciers which pour down from it. The margin of the inland ice lies upon land which has an elevation of 1,000 to 3,000 feet, in consequence of which, as the glaciers pour down from this level to that of the bottoms of the fiords, the crevassing which is caused by their drainage from the inland ice extends for many miles inland, similar to the current in a large body of water when a dam has given way. Small valleys and narrow gorges pass down from the upper levels of Nugsuak peninsula, many containing glaciers, some of which reach the waters of the fiord, but the larger number do not.

Along the margin of the inland ice there are many isolated peaks, nunataks, rising above the level of the ice, sometimes lying wholly within and entirely surrounded by the ice, in other cases lying at the edge of the water and surrounded by varying proportions of ice and water. None of these lie at any great distance within the ice margin, as recorded in other places, no instance being noted of a greater distance than about a mile between these and the main portion of the land.

The Inland Ice. — The edge of the inland ice as it lies upon the Nugsuak peninsula, upon the nunatak between the Great and Little Karajak glaciers and upon Ekinga, the point just northwest of the Itivdliarsuk glaciers, is usually nearly vertical. Its height or thickness varies from 10 feet to about 40 feet. In some cases it is slightly overhanging, in others slightly retreating, but in nearly all cases showing the edge of the layers of which the mass is made up. Here usually the layers are inclined toward the margin. In a few cases the edge has a slope of a sufficiently low inclination to allow it to be ascended, but is always steep.

The marginal area is penetrated on the surface by numerous holes varying in size from the fraction of an inch in diameter up to at least 3 feet, and having an average depth of about 2 feet. At the bottom of each one is a thick layer of dust, which being dark in color causes the hole to appear to be of great depth. So numerous are these where most abundant that they occupy nearly one-half the entire surface. The area occupied by them is perhaps a mile in width, extending inland from the extreme edge of the ice. The same area is very rough and uneven, the surface consisting almost wholly of hillocks or hummocks having much the shape of sand dunes, and an average height of about 2 feet above the bottoms of the hollows between them. This makes the dragging of a heavily-laden sledge difficult, but does not interfere seriously with walking. Beyond this area the surface is comparatively smooth, and it is hard, solid ice throughout. No drifting snow was seen in any portion, this latter fact being due to its being late midsummer.

Between the crevassed areas and the land the ice is traversed by a series of cracks about parallel in direction to the edge of the ice, which have remained partially open till filled with water which has frozen to form a clear transparent vein of new ice with varying widths up to 6 and 8 inches. These larger cracks produce a strong effect upon

the drainage, deflecting the smaller streams for a considerable distance along their own direction and away from the slope of steepest descent toward the land, till the volume of water becomes sufficiently large to break across the little divides between the various cracks. Crossing this series of cracks at a high angle, generally more than 60°, is another series, not so pronounced, however, and seldom showing any trace of vein ice. These two series of cracks divide the ice into large rhombohedral blocks, 10, 20, and more, feet in diameter. The general appearance presented is strongly that of a series of parallel and intersecting joint cracks in slate rocks.

From the highest point reached, having an elevation of 2,960 feet, was obtained a wide range of vision to the south, southeast and north, but to the northeast and bending toward the eastward the surface of the ice rising inland shut off our view within a few miles. The general topography of the inland ice as here seen is rolling and undulating, very similar to the surface of the prairies, and indeed reminds one strongly of them when clothed in snow. The surface of the ice opposite the head of the nunatak between the two Karajak glaciers has an elevation of something less than 2,000 feet. The highest point reached, being 2,960 feet at a distance of about fifteen miles inland, would give an average gradient of about 67 feet per mile, or about 1 in 27. Nansen reports 1 in 23 on the east coast for the same distance.

Numerous small streams abound, flowing with considerable velocity down the steeper slopes or becoming sluggish on the gentler slopes. Those on the slopes directly toward the land fall over the vertical edge of the ice in picturesque little cascades or cut entirely through its thickness, emerging from cañon-like walls. In some of the basin-like depressions the waters from many converging streams collect, forming large, almost marsh-like areas of shallow waters and soft snow. In one case a fine lake of clear water was observed having an area of several acres. The greater number of the streams are small and easily crossed, those serving as outlets for the lakes and marsh-like expanses being the larger, but these are seldom more than 6 or 8 feet broad. The largest stream seen was encountered at the farthest distance inland. This had a width of nearly 20 feet from brink to brink, a depth of nearly 15 feet to the surface of the water, and below that a depth of about 5 feet of water. This river was flowing directly toward the interior, its current having a velocity of three to four miles per hour.

With the exception of the dust found in the dust-holes, no detritus occurs on the surface of the inland ice or in its mass so far as revealed by the numerous crevasses or the cuttings of streams. In the case of the large stream just mentioned the water was absolutely clear and its walls were of clear, transparent, beautifully blue ice, except a thickness of about a foot at the upper surface, which was more or less porous and white. Not a trace of detritus was to be seen in the ice or in the bed of the stream. Along the margin of the inland ice, best seen on the Nugsuak peninsula where there is no tendency toward a lateral movement, there is a well-defined moraine made up of till, which varies in places to sand or clay containing subangular and rounded fragments. This moraine sometimes lies directly against the front of the ice, sometimes is separated from it so as to produce a perfect esker-like ridge. Between the ice and the detached moraine in the latter case, there generally runs a stream of considerable size which has apparently caused the retreat of the ice front by the washing of its current. In only one place was detritus observed upon the ice far enough back from the edge to be entirely detached from the moraine in front. At the extreme upper end of the nunatak lying between the Karajak glaciers, the edge of the ice has a gentle slope sufficiently low to ascend with little difficulty. Here back sometimes a hundred to two hundred feet upon the ice rises the crest of a moraine, but there is no moraine in front. This seemed to be simply the crest of the normal moraine, the snows of the last season or seasons having buried the remainder in a large drift which also forms the gentle slope of the edge. This view is sustained by the fact of the fresher appearance of the surface and its much softer character than elsewhere. There is no indication that this line of detritus is an accumulation of material along the end of a detritus-bearing layer of the ice. No such detritus-bearing layers or zones were anywhere observed in the inland ice or along its edge.

Along the front of the ice in the depressions of the undulating surface of the country are numerous small lakes walled in by vertical or overhanging cliffs of ice on one side, and the basin-like slopes of the country on the other side. The largest of those observed on the Nugsuak peninsula was perhaps a hundred acres in area. This was fed by a large torrential stream flowing for a considerable distance between the moraine and the ice, and formed by the aggregation of a multitude of the small streams flowing off the ice front. At the

mouth of the stream the delta deposits were forming a typical sand plain on a small scale. The lake was not of great depth, probably not over 100 feet, but stood at nearly its highest level, its waters having reached and overflowed the summit of the divide on the side opposite the ice. The outlet stream had cut but a short distance downward, so that the waters had been lowered but little.

Evidently the depression now occupied by this lake had recently been filled by a lobe from the ice front, as well-defined moraines lie around its outer margin, and are cut through by the outlet stream. These lie at different elevations above the lake, so that they could not have been formed by the shoving of the surface ice of the lake, and are highest above the waters near the present ice front. The former extension of the main mass of the ice sheet will be discussed in a later portion of this paper.

The Karajak Glaciers. — The general direction of the Karajak fiord as it penetrates the land is southeasterly. Near its inner extremity it bends nearly at a right angle to the northeast, and there is divided into two unequal arms, one about five miles in width, the other about two miles, which are separated from each other by a long, low nunatak. In these two arms lie the Great and Little Karajak glaciers, which in their direction of flow make an angle of about 45° with the general direction of the front of the inland ice. Of these, the former, taking its rise far back in the ice-cap at least fifteen or more miles from its edge, flows down the fiord nearly to the lower extremity of the nunatak a farther distance of ten or more miles, filling it with a mass of ice which rises nearly to an average of 500 feet above the waters, and presenting to them the ordinary vertical face of a glacier terminating in the sea. Throughout the entire length of its course it is deeply crevassed, its surface consisting of an alternation of pinnacles and crevasses, so that it can not be traversed at any distance from its sides except in a bay-like indentation of the nunatak, where, as Professor Burton has demonstrated by measurements, the ice is stagnant or has little motion, in consequence of which ablation predominates over the crevassing and a comparatively smooth surface results. Here the surface can be traversed for about three-fourths of a mile, or less than one-fourth the entire width of the glacier, and here were made the measurements for the determination of the rate of motion. The elevation of the surface of the inland ice where the glacier passes from it is about 1,500 feet above sea level, giving a

descent between this point and its face of 1,000 feet, an average gradient of 1,000 feet in about ten miles, or of 1 in 52. The gradient of the upper third of its length, however, is much greater than that of the lower two-thirds, much of which is quite low.

The vertical character of the edge of the ice is more marked even in the case of the glaciers than with the ice-cap itself. With the exception of the single place where Professor Burton ascended by means of a moderate slope to the surface of the ablation area, the whole length of the glacier on the side next the nunatak has a nearly vertical face of pure, mostly transparent blue ice, with an average height of 20 feet, and often of 30 and 40 feet. (See Figure 1,[1] where the height is about 20 feet.) Although verticality is the normal rule, the face is sometimes slightly retreating, sometimes slightly overhanging. The cause of this vertical character seems to be partially if not largely due to the lateral streams and waters of the small marginal lakes undermining the edge, blocks from which fall off, the fractures in such cases being nearly vertical. The low angle of the sun's rays may also have a share in the work as discussed by Professor T. C. Chamberlin, for the glaciers farther north. The overhanging faces in many cases are apparently due to a shearing motion of the upper layers over the lower. This was indicated quite strongly in one instance, where a layer projecting slightly beyond the ones above had caught a little detritus as it rolled down. This same ledge continued from the slightly inclined face along a portion of the overhanging face, and here still the detritus remained which had been caught in its descent before the shearing motion had changed this part of the face to an overhanging one. A cavern presented a chance for a study of the material forming the layer upon which the detritus had lodged, and also for several feet above, showing them to be free from detritus and consequently that the detritus could only have come from the upper surface and caught upon the shelf while the face was inclined, and that its present overhanging form was due to the shearing motion of the upper portion of the ice.

The surface of the glacier is much higher toward the central portions than along the margin. No measurements were made to determine this central elevation, but it could not be less than fifty feet above the marginal areas.

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[1] The illustrations used in this article, with a few exceptions, are from photographs made by members of the Boston party, especially by A. M. Dodge.

FIG. 1. THE VERTICAL EDGE OF THE GREAT KARAJAK GLACIER, ABOUT 20 FEET HIGH. THE SMOOTH SURFACE IN THE FOREGROUND IS A PORTION OF THE GLACIER, WHICH IS HELD IN A BAY-LIKE INDENTATION OF THE LAND, HAVING LITTLE OR NO MOTION.

FIG. 2. SMALL ARCH LEFT AFTER FALLING OF MAIN PORTION OF ROOF OF SUB-GLACIAL STREAM NEAR EDGE OF GREAT KARAJAK GLACIER. THE VERTICAL FACE PRODUCED BY THE FALLING OF THE ROOF IS SEEN ON THE LEFT.

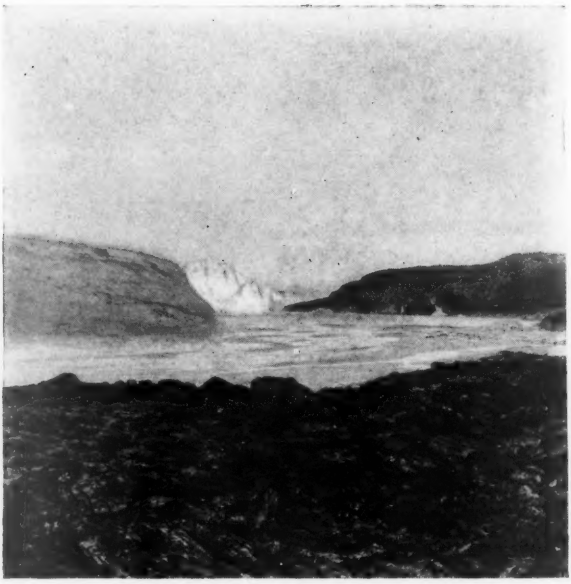

FIG. 3. LATERAL LAKE WITH DELTA AND MEANDERING STREAMS AT THE SIDE OF THE GREAT KARAJAK GLACIER. THE INCLINED EDGE OF THE GLACIER IS SEEN AT THE LEFT. NEAR THIS POINT ASCENT WAS MADE TO THE SURFACE OF THE INLAND ICE.

Numerous small caverns presented an opportunity for the study of the internal structure. In nearly every place so studied the ice presented a moderately well banded appearance evidently due to shearing, with an inclination of the layers toward the center of the glacier at an angle of about 30° in the steepest cases, and varying from that to nearly horizontal. Of the coarser banding or stratification that might be produced by the seasonal accumulation of snows, nothing was seen, though in a few instances broad bands darker than the general mass of the ice were seen, which possibly are due to this cause.

Numerous small streams flow from the surface of the ice toward the land over the uncrevassed area. Falling over the vertical face as in the case of the streams from the ice-cap, they produce a series of very picturesque cascades, one of which attained a sufficient size to have the roar produced by its plunge into the lake below heard at a distance of a half-mile. These small streams give rise to a large lateral stream which in portions of its course flows with torrential velocity. This flows along the margin of the glacier in varying positions, sometimes with the vertical face of the ice forming one of its walls. sometimes bending directly into the ice in caverns of the most beautiful blue color, and disappearing thus for many rods to again reappear and continue its external course; again it winds away from the ice front and flows between the lateral moraine and the land. When near the ice front or within it the bed of the stream is usually of clear, transparent ice through which the stream has not cut to the bed rock, or is often of sands and gravel accumulated upon its ice bed; but toward the lower portion of the glacier the ice for long distances does not rest upon the underlying rock surface, and here the stream flows directly over the bare rock and is broadened out into a thin, much broken sheet of water, which sometimes retreats for a long distance from the edge of the ice below the mass of the glacier. The caverns formed by the stream have usually low broad arches, rendering access within them very dangerous, owing to the falling of fragments from above or the falling of the whole roof, but in a few cases they have narrow high arches perfectly safe for penetration. It is to the falling in of these arches that the vertical face of the ice edge is due in most places. In some cases a small portion of the arch is left, forming a pillar as shown in Figure 2.

In the numerous bay-like indentations of the coast line of the nunatak, the waters of the marginal stream accumulate and swell out into

the expanse of lakes of various sizes up to an area of many acres. The largest of these is near the head of the glacier and not far from the point where ascent to the inland ice was made. This lake is long and comparatively narrow, as are nearly all those which occur alongside of the glaciers. Its greatest length may be something over a half-mile, while its width is nowhere greater than an eighth of a mile. The detritus washed into its upper end by the large stream flowing into it has built a broad fan-like delta, over which the waters of the river meander in a series of small streams. A view of this looking down stream is shown in Figure 3.

The lake next largest in size, shown in Figure 4, is situated about two miles from the front of the glacier. Here a broad valley in the side of the nunatak has permitted the formation of a lake of considerable width, but of which a large part has been drained so that its waters had fallen to a level about 20 feet below their former surface. The upper level of the water is shown by the line of melting seen in the view. Through the large amount of detritus which had accumulated along the margin of the lake, a little stream coming down the valley has cut its way, nearly keeping pace with the fall of the waters of the lake. In the little valley thus cut through the detritus, as well as along the whole margin, wave marks showing the successive stages of water level are well shown, as seen in Figure 5.

Over the larger proportion of the surface of the glacier there is no detritus to be seen. The pinnacled area is very clear and white as far as it could be seen on the surface, and the crevasses show clear transparent blue ice with scarcely a trace of detritus of any size, except within a very short distance of the sides next the land. On the surface of the bay-like expanse where melting has predominated over crevassing, sufficient dust has accumulated to cause the whole area to have a dirty color, but excepting directly along the margin there are no fragments, and only an occasional pebble or bowlder. The marginal ice as seen by means of the caverns and tunnels along the streams is extremely transparent, so that fragments of rock within it could be seen when imbedded from 8 inches to a foot from the surface. In no place would the detritus held in the ice amount to more than 1 per cent. of the total mass. The great tenacity of the ice is shown in some cases where large fragments held by it project downward from the roof of a cavern. This was shown in one case by a bowlder fully 3 feet in length, of which only a third was held in the

FIG. 4. LATERAL LAKE, IN WHICH THE WATER HAS RECENTLY FALLEN FROM A LEVEL ABOUT 20 FEET ABOVE THE PRESENT ONE. THE VERTICAL EDGE IS ALSO SHOWN AND THE SLOPE OF THE SURFACE FROM THE CENTRAL PORTIONS OF THE GLACIER TOWARD THE SIDE.

FIG. 5. WAVE MARKS FORMED AT SUCCESSIVE LEVELS OF THE RETREATING LATERAL LAKE, AND THE CUTTING MADE BY A SMALL STREAM IN ITS OWN DELTA AS THE LAKE SUBSIDED.

ice, while the remaining two-thirds projected at such an angle as to produce a very strong leverage (see Figure 6), yet it was immovable by any ordinary force that could be applied by the hands.

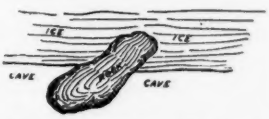

FIG. 6. BOWLDER THREE FEET LONG, HELD SUSPENDED FROM THE ROOF OF AN ICE CAVE, SHOWING THE EXTREME TENACITY OF THE ICE.

A lateral moraine of considerable size, reaching a height in some cases of about 30 feet, borders this glacier throughout its whole length along the nunatak. The material of which it is composed consists of fragments of all sizes up to blocks having a diameter of 20 feet, and of finer gravels, sands, and silt. The larger blocks are almost entirely angular, but the medium in size and smaller ones are mostly rounded or subangular, more often the former shape. A coating of very fine silt covers all the surface from which the water has recently receded as the lakes drained. The outline of the moraine differs considerably in various places. In some places it is a very distinct ridge entirely detached from the margin of the ice, and often then the lateral stream flows between it and the ice. In other cases it lies directly against the side of the ice, and then the lateral stream commonly flows between the moraine and the slope of the nunatak. In still other cases it is very irregular in outline, and perhaps divided into two prominent ridges. In either of these latter cases the irregularity seems to be due to the morainal matter having covered some of the ice, causing irregular melting, as shown in Figure 7.

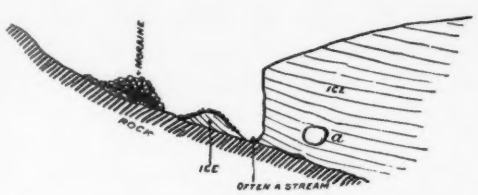

FIG. 7. RELATION OF LATERAL STREAMS AND MORAINES TO MARGIN OF GLACIER. *a*, CAVERN IN WHICH FLOWS AN ENGLACIAL STREAM WHICH MAY BE LATERAL ABOVE AND BELOW THE POSITION OF THE SECTION SHOWN.

The ice front standing 500 feet above the waters of the fiord, strongly crevassed and pinnacled, shows no trace of detritus except in the portions immediately marginal to the land. Owing to the danger from the waves produced by the fragments falling from above, or the large masses rising from below, the ice front can never be very closely approached excepting on the land, and from here, so far as could be seen with a good glass, the main mass of the ice is of that clear transparent blue and green that is seen

in the inland ice especially. In the broader crevasses a mass of ice debris from the falling pinnacles collects to form a conglomeritic or a brecciated structure presenting a very peculiar appearance when detached and floating away as a berg, especially when making up the larger portion of the berg, as it sometimes does.

No streams of any size flow from the front of the glacier above water level, and those that must flow from it below the surface give little indication of their presence — a fact which is probably due to the very numerous fragments of all sizes which completely cover the surface for a long distance from the ice front, and also that the surface is in a state of almost constant commotion from the falling fragments. Small fragments are constantly falling with reports like pistol or rifle shots, and larger blocks quite often, the latter producing a series of

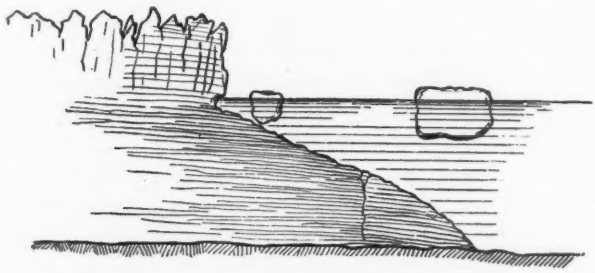

FIG. 8. IDEAL SECTION OF THE END OF THE KARAJAK OR ITIVDLIARSUK GLACIERS.

reports like artillery, and occasionally the whole face for a long distance falls with a tremendous roar that reverberates from the sides of the fiords. Bergs so produced are sometimes a hundred or more feet in length, but never very high above the surface of the water, as the fall and the plunge into the water produce shocks which cause the large mass to crack into numerous small fragments, sometimes entirely to a mass of small debris. The larger bergs are produced by the masses derived from below which are not numerously filled with cracks originally, and are not subjected to the impact with the water surface, as is the case with the falling masses. These masses are detached from that portion of the glacier which projects from the vertical face of the ice out under the water, and seldom detaches any portion of this face in their own evolution.

The face of the glacier has an outline as in Figure 8, produced by

the breaking off of the face of the glacier above water level by the action of gravity on this highly crevassed portion of the mass, and the constant motion of the glacier itself.[1] The lower portion of the glacier is less crevassed than the upper, is consequently more coherent, and is not affected by the sun's rays or by gravity. As the upper portion arriving at the front breaks off and floats away, the under portion passes out under the water until its buoyancy, increased by loss of the weight of that part that has broken away above, becomes sufficiently great to cause it to crack off in very large masses and rise to the surface. All the larger bergs observed in the process of formation had this latter origin. They form immense masses having great length and breadth, and rising high above the water, probably to a hundred feet or more. In rising to the surface they produce dull, grumbling roars, and a much greater disturbance of the water than in the case of the blocks falling from above, as the disturbing power is deeper seated. Although the radiating waves may have died out on the surface before the shore is reached, so that scarcely a gentle swell is noticed, yet the water along the beach is churned into foam and spray dashed into the air a hundred feet in height. Before coming to rest the bergs must seek a position of stable equilibrium, and before reaching this they vibrate up and down or revolve over many times, and often end by the side that rested upon the bottom of the fiord being largely or completely exposed to view. These bottom surfaces are smooth and often broadly grooved, but with the exception of those formed near the shore, free from all detritus, consisting of pure ice with no discoloration. This implies that the glacier is moving over a bottom of the fiord that has been cleared of all detritus, that has become completely smoothed into probably roches moutonnées surfaces, and that the glacier contains no detritus in its mass. Occasionally bergs are seen that are highly discolored with mud or silt, but these, so far as observed, are detached from portions near the margin where the detritus has been carried into the ice by the lateral streams. That animal life is abundant in the waters directly in front of the ice is indicated, as already shown by Professor Tarr, by the gathering of

[1] Since the above was written, I. C. Russell's "Glaciers of North America" has been published, in which the author advocates the same view as here adopted. See pp. 85, 86 of that work. Reid's view as presented by Russell, p. 85, would be untenable for the phenomena as seen in Greenland.

the gulls about every point of disturbance to reap a harvest of food from the waves.

Professor Burton has demonstrated that at the place where he measured the movement of the glacier there is an area which has little motion, and even that nearest shore being slightly up stream, while at a distance of about two-thirds of a mile from shore there is a motion at the rate of 2.42 feet per day, and at a distance of 2.4 miles, a rate of about 19 feet per day. In the region of slow motion along the margin where bowlders and ledges project upward into the lower portion of the ice, the latter after having passed over the obstruction has flowed backward and downward so as to keep in contact with the lee side of the rock, producing a fan-shaped structure in so doing, as shown in Figure 9. In cases where the motion is more rapid, the same tendency to flow backward and downward with the fan-like structure is marked, but this latter motion has not been able to keep pace with the movement of the glacier, and a vacant space is left on the lee side of the rock, as shown in Figure 10. In this particular case of Figure 10 there would seem to be a variation in the ratio of the two movements, as the fan structure shows an exact mold of the surface of the rock in its outline, thus indicating that this structure had at one time kept pace with the movement of the glacier, but that more lately the latter movement had increased its rapidity. In those portions where the motion is most rapid the open space on the lee of the projection is very large, and there seems often to be no tendency of the ice to flow downward or back-

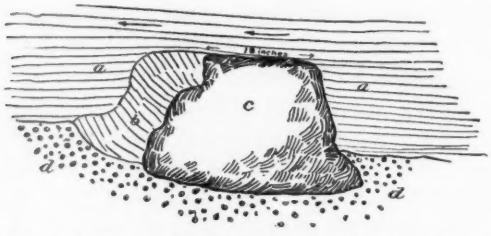

FIG. 9. GLACIER, *a a*, FLOWING OVER PROJECTING POINT OF ROCK, *c*, WITH ICE, *b*, FLOWING DOWNWARD AND BACKWARD TO FILL SPACE LEFT VACANT BY THE MOTION. DIRECTION OF MOTION FROM RIGHT TO LEFT.

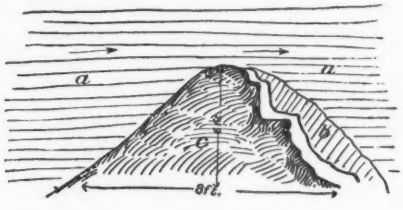

FIG. 10.

FIG. 12. THE SMALLER KARAJAK GLACIER.

FIG. 14. CLIFF FACE OF THE ITIVDLIARSUK GLACIER. HEIGHT ABOUT 200–250 FEET ABOVE WATER LEVEL, IN HIGHEST PORTIONS. (*From National Geographic Magazine.*)

ward to fill this space (see Figure 11), while in other cases there is a large area of the radiate structure.

In the region of least motion, generally, the laminæ of the ice on the stoss side end directly against the projecting boss of rock, with no tendency to curve upward and pass over the obstruction (Figures 9 and 10), while in the region of more rapid motion the laminæ very commonly curve upward and tend to pass above (Figure 11). In no case, however, was any detritus observed passing upward into the ice by means of this slight upward flow.

In the region of greatest motion near the end of the glacier, the ice does not rest upon the rock except upon the larger projections of the bed, leaving large areas of open space between the bottom of the ice and the rock surface below, so that one can often penetrate to a hundred or two hundred feet beneath the mass. The height of this open space varies up to a maximum of about 3 feet, but is usually not more than 1 or 2 feet. Bowlders in this portion are rolled along without being frozen in the ice, and, striking the projecting bosses of rock, are sometimes fractured, sometimes splinter off portions of the boss; smaller bowlders caught between the bosses and the larger bowlders are often crushed to powder.

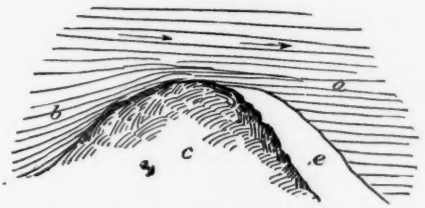

FIG. 11. UPWARD BENDING OF ICE LAYERS ON STOSS SIDE OF PROJECTING ROCK, SEEN ONLY IN CASES OF RAPID MOTION.

Smaller Karajak Glacier. — From a well chosen position on the nunatak between the two Karajak glaciers, the smaller one can be seen throughout its whole extent from the point where the crevassing of its current first begins in the inland ice to its frontal face. It is not more than two miles in width nor over five or six miles in length. Its gradient is high (see Figure 12), and consequently it is strongly crevassed in every portion. On the western side there is a large continuous moraine, but on the eastern side a portion of the nunatak bordering the glacier is very precipitous, and at the foot of these vertical faces there is no moraine. Along the less precipitous portions and the valleys there are well-defined moraines. No position was obtained where the height of the frontal face of this glacier could be estimated.

Itivdliarsuk Glaciers. — About twenty miles north of, and parallel

to, Karajak fiord is the Itivdliarsuk fiord, which is a little less in size than the former. Between the two is the Sermilik fiord, still smaller than either. The glacier at the head of the latter fiord was not visited, though numerous bergs were passed which were derived from it.

At the head of Itivdliarsuk fiord its waters abut directly against the high point of land which lies between this and Sermilik fiord. Along the northeastern side, just below the end of the fiord, a broad area of ice enters the water. This is divided by long narrow nunataks into smaller glaciers, two of which are shown in Figure 13. The lower one, shown in Figure 13, was visited, while the upper was only seen from a distance. The width of this as it enters the water is about three miles, while the height of its face is not above 250 feet. The broad deep crevasses often reach down to the level of the water, producing the effect of a much more irregular frontal face than in the glacier previously observed. (See Figure 14.) Its entire length is about the same as that of the Greater Karajak, and its width is nearly constant throughout, about three miles.

The average gradient of this glacier is a little less than that of the Karajak, probably, but no determinations were made. Not far from the front the central surface is slightly higher than it is a half-mile farther up stream. The whole surface is strongly crevassed, even to the shore. (See Figure 15.) The sides are less steep and high, however, and the glacier was entered for a distance of a few rods at several places, though at no place could any great distance be reached. The vertical character of the edge of the ice is not so marked in this glacier, but it is still prominent. Marginal streams and lakes are also characteristic, and several large lakes on the surface, winding in and out among the pinnacles, were seen from a position overlooking the full width of the glacier. No distinct surface streams were seen.

The general mass of the ice here is also free from detritus. A lateral moraine holds the same relations as before, and is extremely till-like in its characteristics, containing a very large amount of fine silt, and when dry having the tenacity of till. A marginal strip of the ice a few rods in width is highly discolored with fine detritus, but only a few fragments are present. Near the head of the glacier and not far from its center lies a small nunatak, which gives rise to a medial moraine as seen in Figures 16 and 17.

Figures 16, 17, and 18 present views of the inland ice and the upper portions of the Itivdliarsuk glacier as seen from Mt. Ekinga

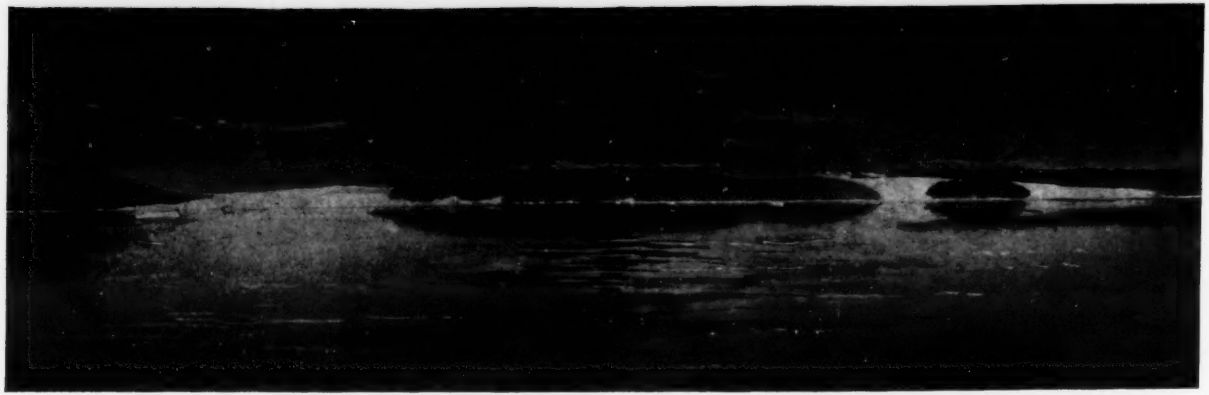

FIG. 13. THE ITIVDLIARSUK GLACIERS AND NUNATAKS FROM THE OPPOSITE SIDE OF THE FIORD. THE SUMMIT OF EKINGA IS SEEN UPON THE LEFT.

FIG. 15. CHARACTERISTIC SURFACE OF THE ITIVDLIARSUK GLACIER.

just northwest of the glacier. Figure 16 shows the upper third of this glacier and portions of the nunataks and glaciers to the southeast. The direction of the motion is shown by the medial moraine proceeding from the small nunatak. The greater volume of the ice comes from the eastward of the nunatak, and striking the stream from the west, deflects it strongly westward into the bay-like indentation on the Ekinga side. Figure 17 gives a nearer view of the small nunatak, and presents just the upper point of the larger nunatak on the east side. This view shows a strong difference in the crevassing of different portions of the glacier. The western side of the portion which flows between the nunatak and Ekinga is highly crevassed and pinnacled, while the eastern portion is comparatively much smoother. Again the lateral portions of that part which flows from the east of the nunatak are highly crevassed and pinnacled, while the central portions are smoother. These conditions continue nearly to the front of the glacier. Figure 18 presents the extreme upper portion of the glacier between the small nunatak and Ekinga and the edge of the inland ice stretching away to the northward. The head of the Ignerit glacier is seen between Ekinga and the first high peak beyond, from which a long, sharp spur projects into the ice. These three views all show the surface of the inland ice looking over its vast undulating expanse toward the interior. The horizon line of ice and sky is considerably higher, $2\frac{1}{2}$ degrees, than that of our point of view, presenting a fair view of the gradient of the marginal area of the ice sheet. Figure 19 is taken from nearly the same point as the three preceding on the summit of Ekinga, 3,100 feet altitude, but looking northward and showing the edge of the inland ice as it lies upon the highlands north of the region shown in Figure 20. The ice on the extreme left as here seen is a portion of the local ice-cap lying upon the large peninsula northeast of Agpat island.

The Small Valley Glaciers and Glacial Tongues or Lobes.—Into the small valleys and gulches which are sharply incised in the northeast side of the Nugsuak peninsula, flow streams of ice from the local ice-cap of Nugsuak, some of which are broad, rounded lobes, some long, narrow ribbons of ice, while others present the ordinary forms of valley glaciers, which in a few cases reach down to sea level and enter the waters of the fiords. Nearly all these small valleys and gulches are practically at right angles to the general trend of the coast. Ekaluit and Itivdlek are larger valleys which are exceptions, their trends being

at sharp angles with the coast. The latter of these contains no glaciers; the former has several which occupy small lateral valleys, but none of which reach the bottom of the larger Ekaluit valley itself. As the Nugsuak ice-cap lies wholly upon the south of this valley, the glaciers are necessarily limited to that side also. The two nearest the mouth of the valley only were visited, but several others were seen with the glass. Figure 20 presents a view but a short distance up stream, and shows the first two glaciers which were visited, and also a small detached portion of the ice-cap at the upper right of the view. The first glacier is fully shown in the view at a distance of an hour's walk. It plunges down through a narrow cañon with nearly vertical walls in its upper portion, with a very steep angle of descent which was not measured; then broadening, it loses its steepness and flows for a considerable distance with an average inclination of about 12°, and then changing again to an angle of 28°, which rises to 35° at its terminus. The upper and lower portions are strongly crevassed, but the middle portions only slightly so. The central portion throughout its whole length is much higher than the sides, the inclination of the surface from the sides toward the center being sufficient to make a pike staff necessary for the ascent. Small surface streams flow from the uncrevassed portions, forming large lateral streams.

As seen in the view the lateral moraines are very broad toward the lower end. These lie almost wholly upon the ice, and the stream flows between them and the pure ice of the glacier. The moraines thus appear to be much larger than they really are. As the stream cuts down into the ice, the detritus falls into it on the side next the moraine, the result being that the stream constantly encroaches upon the glacier as the detritus constantly follows and pushes it. Toward the upper end where the cliffs are nearly vertical, there are no moraines. At the foot there is a very definite fresh terminal moraine. The ice of the glacier is firm, compact, and blue in color. The largest stream from this glacier bursts directly from the foot of the ice and plunges down the valley in a torrent. The detritus borne by this stream has been spread out in a broad, low, alluvial fan over which its waters flow by several divergent branches. The glacier seen to the left in Figure 20 is not connected with the ice-cap above, but flows from a large cirque which is indistinctly seen in the view. On the left the walls are steep and high, on the right they retreat, the narrow valley above becoming broad and cirque-like. A portion of the glacier flows around

FIG. 18. THE INLAND ICE AS SEEN FROM THE SUMMIT OF EKINGA, 3100 FEET ABOVE SEA LEVEL, WITH A PORTION OF THE LOWER ITIVDLIARSUK GLACIER AT THE RIGHT.

FIG. 20. EKALUIT VALLEY WITH TWO OF ITS LATERAL VALLEY GLACIERS, AND A PORTION OF THE LOCAL ICE-CAP OF NUGSUAK PENINSULA AT THE RIGHT.

a projecting boss of rock into this cirque, giving rise to a series of very highly contorted flowage lines much resembling those of basic lavas. In its first plunge from the cirque the glacier has an inclination of 55°, this area having large crevasses; the inclination then changing to 30° till broadening out to the wide expanse below, it has an inclination of 18° with very slight amount of crevassing. The lower portion very much resembles a huge shell in shape with the edge of the layers simulating its lines of growth. Lateral moraines are not so marked here, but the terminal moraine is very large and characterized by many very angular large blocks. The end of the glacier rests directly upon the fresh bare rock which forms the bottom of the valley for a short distance below the foot of the ice, then becoming steeper it sinks below a large mass of detritus and entirely disappears. The lateral stream upon the left side is of moderate size, that on the right is lost below the ice and in the detritus; these uniting flow down the valley in a foaming torrent which has cut into the detritus above mentioned a depth of at least 30 to 40 feet without reaching the rock below. A broad, low, alluvial fan is also formed by this stream. The streams from the various glaciers unite to form the Ekaluit River flowing through the large valley, which in some portions of its course is torrential; in others it broadens out into small lakes in which the water is so clear that the bottom can be plainly seen for long distances from the shore.

The long narrow strip of ice seen at the upper right of Figure 20 is a part of the ice-cap which has been separated from the main mass by the melting of the thinner portion lying upon the sharp edge of the plateau. The inclination of its surface is about 25°. Its material is hard, compact, firm ice. Along its lower edge, which has an altitude of about 3,000 feet above sea level, is an exceptionally fine example of a bowlder pavement, the angular fragments being well fitted together, and the surface being nearly as smooth as an ordinary street pavement. This was visited during a heavy rain, so that there was no opportunity to obtain photographs of it.

From Ekaluit to the neighborhood of Sermiarsut the small glaciers are very numerous, some reaching sea level, but the majority only approaching it. In one case a little valley reaching back to the ice-cap ends with a nearly vertical face. Over this face the ice falls in large blocks which break into fragments and melt at its foot without accumulating to any amount. The vertical face of ice so produced above the valley shows its thickness to be at least 100 feet. Few of

these lateral valleys show any cutting below sea level, the streams from the glaciers coming with torrential velocity to their mouths at the margin of the fiord. In a very few cases the waters of the fiord enter the valleys for a very short distance. A small glacier only a few hundred feet in width at most enters the water in one of these valley inlets.

Just east of Sermiarsut two large glaciers from closely adjacent valleys not only reach sea level, but push out slightly into the fiord. A very large amount of detritus has been brought down by the streams and by the ice, which the glaciers now seem to be over-riding as though advancing with considerable rapidity. The gradient of both these glaciers is very low as compared with all others of the valley glaciers. Extremely large lateral moraines come nearly to the very front of the glaciers, passing out a few hundred feet beyond the fiord margin.

On the island of Disko local ice-caps cover portions of the surface, which is a plateau of varying elevation, seldom below 2,000 feet above sea level and rising in summits to between 5,000 and 6,000 feet. Just back of Godhavn the ice-cap comes almost to the edge of the vertical basalt cliffs. The character of this small ice-cap is very different in many ways from that of the main inland ice. Its edge is extremely attenuated, the margin often decreasing to the thickness of an inch or less, and small detached portions being found at a distance of a few rods from the main mass. The ice rises to a broad dome-shaped form with an average inclination of about 13° on the surface. The surface was soft at this time, August 3, making walking quite difficult, each step plunging to the depth of six inches to a foot. Occasional narrow crevasses occur not over a foot in width, indicating a general motion of the whole mass. West of Godhavn a glacier plunges sharply down a narrow valley. The source of this glacier in the ice-cap is surrounded by crevasses which do not extend far back. Northeast of Godhavn is Blase Dale, whose glaciers have been described by Chamberlin and Salisbury. Here again the crevassing at their source extends back but a short distance into the ice-cap. This feature is in marked contrast to the great extent of the crevassed areas seen at the source of the Karajak and Itivdliarsuk glaciers. The lack of crevassing here is due to the smaller size of the ice stream, and to its sharper precipitous plunge. It is of interest to note that a spring similar in all characteristics to the one described and figured by Salisbury[1] was found in strong action on the same glacier.

[1] Journal of Geology, IV, 1896, p. 809.

FIG. 16. THE UPPER PORTIONS OF THE ITIVDLIARSUK GLACIERS WITH INLAND ICE IN THE BACKGROUND. THIS VIEW IS TAKEN FROM THE SUMMIT OF EKINGA, 3100 FEET ABOVE SEA LEVEL.

FIG. 17. UPPER PORTION OF LOWER ITIVDLIARSUK GLACIER WITH SMALL NUNATAK IN CENTER AND THE INLAND ICE IN THE BACKGROUND. TAKEN FROM THE SIDE OF EKINGA.

The Former Extension of the Ice. — With one or two exceptions all the large valley glaciers, whether terminating in the sea or on the land, and the glacial lobes, exhibit evidences of diminution in size recently, and also during a long period of time. At Sermiarsut, nearly opposite the village of Umanak on the Nugsuak peninsula, the two glaciers as described above have brought down a large amount of detritus, and have projected it into the fiord. The material is all fresh, not covered with lichens, and the ice is apparently overriding the portions deposited in front, passing out over it for some little distance beyond the walls of the fiord, but not entering the water, the detritus bringing it above sea level. Nowhere else was any evidence of advance observed, and this may be more apparent than real.

Along the sides of the valley of the Greater Karajak glaciers at an elevation of from 60 to 100 feet above the present lateral moraine, there is a definite surface at most points but obliterated in others, marking a former level of the glacier at this height. Below this level the valley's sides are well covered with morainal material, all of which is well covered with black lichens except the material of the present moraine. This upper line can be traced along the whole length of the valley from below the present terminal face of the ice to the head of the nunatak, holding approximately the same elevation above the present moraine throughout its course.

At the Itivdliarsuk glacier the point of land forming its western boundary rises at Ekinga to an elevation of over 3,000 feet. At the southwest it is very precipitous from the summit downward till within a height of 400 or 500 feet above the sea, sending out here a long low point of about this elevation. Through this point runs a small valley diverging from the large Itivdliarsuk at nearly a right angle. At a time when the Itivdliarsuk glacier was larger than at present, it sent a branch down through this valley which entered the sea two or three miles farther down the fiord than the main mass. The upper surface of the lateral moraines formed on either side of this branch are well preserved. This moraine is also more or less continuous up the large Itivdliarsuk valley toward the inland ice.

In the valley of the Ekaluit there is no glacier to-day, except the small ones in the lateral valleys leading into it, but there is good evidence of its being formerly occupied by one. The northeast side of the valley is precipitous, nearly vertical in places, so that old moraines cannot be traced easily on that side, though at the foot of the preci-

pices there is a general mingling in large masses of fresh talus and old morainal material, the latter being mostly buried, however. The southwest side of the valley has a comparatively gentle slope, and here the old moraines are well preserved. In some places the whole side of the valley from the bottom upward to the foot of the more precipitous upper portions is covered with a morainal sheet of considerable thickness. The streams issuing from the small lateral glaciers cut downward through this sheet, sometimes reaching its lower surface, sometimes not; in one case cutting to a depth of at least 30 to 40 feet without reaching the underlying rock. The material of this thick sheet is a mixture of rolled or rounded and subangular fragments with some finer material. Lying more especially at the mouths of the lateral valleys, it may have been derived partly from the terminal moraines of the small lateral glaciers when they extended farther than to-day. Between the valleys the underlying rock is often at the surface.

Running along the sides of the valley at a considerable elevation above its bottom are three very distinct and parallel old lateral moraines. These are practically continuous as far as the valley was seen, except where cut by the lateral valleys and glaciers, as was done in each case. All three moraines are sharp, well-defined ridges, maintaining about the same elevation above the valley bottom all along its course, dropping as it drops as they approach the sea. The altitude of the crest of the lower one of the three just seaward of the first lateral glacier is about 875 feet above sea, and 500 to 600 feet above the valley bottom. The altitude of the second, 1,065, and of the third about 1,150 feet above the sea, or more than 700 and 800 feet above the valley bottom. The depth of the glacier that formed this upper moraine must have been nearly or quite 1,000 feet. The foot of the present small glacier in the first lateral valley reaches down to a point about 1,125 feet above the sea, just a few feet lower than the crest of this old moraine. The sides of the lateral valley, where not too steep, also have old lateral moraines nearly 100 feet above the present glacier, and distinct old terminal moraines occur at some distance down the valley below the present ice front.

In the Itivdlek valley bowlder pavements and lateral moraines were seen, but no observations were made upon them. Back from this valley the edge of the inland ice at Iterpiluak gives evidence of having quite recently sent a lobe downward to occupy a small shallow depression which is now occupied by a lake. A moraine extends from the

present moraine of the ice front around the lake at a slight elevation above its present surface. The larger portion of the material of this moraine is not covered by the black lichens which cover all old detritus, showing that it is comparatively of fresh formation.

Many of the small glaciers along the Nugsuak coast from Ekaluit to Sermiarsut have large moraines lining the valley sides high above their present level and large enough to be distinctly seen from the sea. Large terminal moraines can also be seen in most of the valleys far below the present ice front. The valley bottoms are sometimes well rounded, and in some cases the present stream has cut a gorge down into the rounded floor, as shown in Figure 21.

Along the more or less precipitous sides of the Nugsuak peninsula are occasional pieces of old moraines at various altitudes, some well defined, some indistinct. The same is to be seen on the north side of Storöen, especially well at an elevation of between 500 and 1,000 feet above sea level.

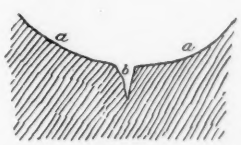

FIG. 21. ROUNDED BOTTOM, *a a*, OF OLD VALLEYS ALONG NUGSUAK PENINSULA, WITH GORGE, *b*, CUT BY STREAM FROM THE RETREATING GLACIER.

The islands of Umanak and Ikerasak present types of topography essentially like each other, but differing from all others in this region. Each consists principally of a moderately low surface, not much if any over 1,000 feet in altitude in its highest portions, well glaciated, and rounded into roches moutonnées. Resting upon this smooth and comparatively broad pedestal rises a sharply angular elongated peak with almost vertical sides and ends too steep for ascent. No smoothing is apparent above the level of the roches moutonnées; all outline above is due to the effect of frost action. The contrast is exceedingly marked and striking. The summit in each case is sharply serrate, but in the case of Umanak when seen from one position a single cleft with rounded peaks on each side form an outline resembling the lobes of a heart; whence the Eskimo name *Uma*, heart, *nak*, like.

The peak of Umanak rises to a height of 3,720 feet above sea level, and that of Ikerasak to 2,550 feet. At the foot of each peak lies a lateral moraine distinctly marked in each case, but better seen at Umanak. Figure 22 shows the southwestern side of Umanak Peak with the moraine at its base. The sharp contrast between the smooth-flowing outlines below and the angular above is also well brought out in this view. Figure 23 is a view of the peak of Ikerasak showing the

contrast in topography again, but not sufficiently near to show the moraine at its base. The material forming the ridge seen in Figure 22 is of true morainal character, angular, sub-angular, rounded and striated fragments with considerable sand and clay intermixed. It is entirely distinct from the talus which is mostly accumulated on the steep lower slopes of the peak, though many fragments of the latter have reached and lie upon the moraine. Most of the morainal material is covered with black lichens, while the talus material is largely free from them and fresher in appearance. There is no soil except in sheltered nooks on these low islands, and practically none at all on the higher islands and peninsulas. Finer drift material is also very scarce, but coarser drift is common. The low main portions of Umanak and Ikerasak islands are liberally sprinkled with bowlders of all sizes up to many tons in weight, and in many cases they almost entirely cover the surface.

Gneiss and schists, cut by granite and by numerous dikes of very dark color and high specific gravity, are the only rocks which form those portions of the mainland, of Nugsuak peninsula and of the islands which we visited. The rocks of later age occur farther westward on Nugsuak. The gneiss is very highly folded and contorted. Figure 22 shows indistinctly an overturned fold. All of these rocks are abundantly represented among the bowlders on Umanak and Ikerasak, but no other variety is found. The bowlders of eruptive rock often show a large amount of decomposition which has taken place since their transportation, some having crumbled entirely, while in others the polished glaciated surface has remained as a shell from which a portion of the remainder has fallen away. The dikes show no greater erosion by glacial action than the inclosing gneiss, being ground and polished to the same level only, not below that of the gneiss, indicating no great preglacial decomposition below their present surface. The surface of the gneiss, while not generally retaining the polish and finer striæ of glacial action, does retain the coarser striæ and grooves, disintegration having taken place but slightly since the disappearance of the ice.

On the nunatak between the two Karajak glaciers, the same conditions prevail as upon the lower portions of Umanak and Ikerasak. Its form is long and narrow, lying parallel to the Great Karajak glacier, and nearly at right angles to the Karajak fiord. Its highest summit on the north is 2,245 feet, and the one on the south a little

FIG. 19. THE EDGE OF THE INLAND ICE AS SEEN TO THE NORTHWARD FROM THE SUMMIT OF EKINGA.
UPON THE EXTREME LEFT IS THE LOCAL ICE-CAP OF AGPAT ISLAND.

lower. The most typical roches moutonnées occur on both summits with striæ and grooves well preserved, and in some cases the polish is also well preserved, but this is exceptional. The direction of motion here was transverse to that of the present Karajak glacier and to the axis of the nunatak, and nearly parallel to Karajak fiord and the Nugsuak peninsula. Bowlders are numerous over all parts of the nunatak. So far as seen they are entirely of gneiss, schist, granite, and the same dark eruptive rock as before noted. Here the eruptive rock, though showing much decomposition in the bowlders, has resisted erosion more strongly than the inclosing gneiss, so that the dikes and intrusive areas sometimes stand out in relief above the gneiss. This is most strikingly shown in an exceptionally fine high roche moutonnée, which consists entirely of the eruptive rock with the polish and striæ well preserved on the stoss side, where it is 10 to 12 feet high and 8 to 10 feet broad. On the Nugsuak peninsula between Itivdlek and Iterpiluak the same general characteristics are found. The bearings of the striæ are quite constantly about parallel to the direction of Karajak fiord, with some local variations.

On the point Anat just west of the Itivdliarsuk glacier, Mt. Ekinga rises to a height of about 3,100 feet. Its summit is finely glaciated, showing some of the finest roche moutonnées. The striæ and grooves are well preserved, and in some cases the polish still remains. Here again the direction of the motion was nearly or quite parallel to the direction of the Itivdliarsuk fiord, and consequently to Karajak fiord. This shows a parallel movement over an area of many miles in width in this region.

Bowlders of gneiss, schist, granite, and the dark eruptive rock are scattered in profusion over the lower portion and to the highest summit. In addition, all over this little peninsula, so far as seen, there are bowlders of limestone slightly fossiliferous, and of a red sandstone that so closely resembles the triassic sandstone of the Connecticut Valley that it could easily be mistaken for it. Neither of these rocks occur in place so far as a careful search would reveal. Their presence here as bowlders implies that they do occur in place farther inland under the present ice-cap. The fragments vary in size all the way from small pebbles to bowlders of 5 or 6 feet in diameter.

Ekinga was the highest point actually reached, and this in common with all lower ones gives the strongest evidence of having been overridden by the ice. From this and other high points, as well as from

the surface of the inland ice, transverse and rear views of higher peaks were obtained. The general topographical appearance changes as much on a large scale according to the point of view from which it is seen, as does one of our smaller roches moutonnées areas when seen from the stoss or lee sides, or transversely.

Seen from the west, and especially when seen from the water level, the general appearance is of strong angularity, so sharply marked in many cases that it is difficult to believe that it has been overridden and smoothed by ice action. Seen from the south or north, and better when viewed from a moderate elevation, the same region shows usually a sharply serrate character, but with the peaks shaped something like the teeth of a circular saw, steep and precipitous on the west, and with moderate or gentle slopes on the east. This characteristic outline so strongly marked over the whole region is well shown in Figure 24, which is a sketch of a portion of the summit of Agpat Island as seen from the water to the southward between Agpat and Storöen.

FIG. 24. SKETCH OF A PORTION OF THE SUMMIT OF AGPAT ISLAND SEEN FROM THE SOUTHWARD, SHOWING THE MARKED STOSS SIDE OF SUMMITS, WHICH APPEAR ANGULAR FROM THE WESTWARD. *a a*, LOCAL ICE-CAP.

Again when seen from the ice-cap to the east, the angularity very largely disappears, if not entirely so. The Nugsuak peninsula, as seen from the highest points we reached to the east and northeast, presents these features so strongly as to convince the writer that its surface even to its extreme outer point has been overridden, and that the ice must have passed into the waters beyond. Professor Angelo Heilprin informs the writer that he has been upon the summit of the outer end of Nugsuak, and finds that it has been glaciated to its extremity. As seen from Ekinga, Agpat presents a very smooth surface with well-rounded outlines, as does all the remainder of the region except the distant and sharp peaks of Upernavik and Ubekyendt islands, the higher points of the former rising to 6,850 feet, being the greatest elevation attained by any summits in this region. Of all the peaks seen in Greenland, these only left a doubt in the mind of the writer as to their having once been covered by the former extension of the inland ice. Professor Heilprin states that he has also been upon the

FIG. 22. THE SOUTHWESTERN SIDE OF UMANAK ISLAND, SHOWING THE MARKED CONTRAST BETWEEN THE ANGULARITY OF THE PEAK AND THE ROUNDED OUTLINE OF THE LOWER AND MAIN PORTION OF THE ISLAND. A VERY LARGE LATERAL MORAINE IS SEEN AT THE LEFT OF THE FOOT OF THE PEAK. A WELL-DEFINED OVERTURNED FOLD IN THE GNEISS IS SHOWN ON THE SIDE OF THE PEAK. THE PEAK IS BELIEVED TO HAVE BEEN BURIED BY THE GREATEST EXTENSION OF THE ICE, AND ITS PRESENT ANGULARITY TO BE DUE TO LATER FROST ACTION.

summits of both Upernavik and Ubekyendt islands and found abundant traces of glaciation produced by ice moving westward.

There seems to be strong and abundant evidence that all of the Umanak fiord region south of the Svarten Huk peninsula which was not seen, and consequently can not be discussed, has been covered even above the highest summits with a sheet of ice having a definite and strong motion in a direction something north of west, about parallel to the general trend of the Nugsuak peninsula and of the Karajak and Itivdliarsuk fiords. That the motion must have been a strong one is shown by the considerable amount of rounding that has been impressed upon the topography even to the outermost points, and by the fact that a large proportion of the bowlders so numerously scattered everywhere show evidence of transportation for a considerable distance in their often well-rounded character. It is unfortunate that the underlying rocks of the region have the same general characteristics throughout, preventing the determination of the distance of transportation. The only exceptions to this are the sandstone and limestone bowlders found at Ekinga, and the lateness of the season when these were found prevented the determination of the limit of their distribution westward. That the ice passed over the summits of Upernavik island nearly 7,000 feet above the sea indicates a great thickness at its maximum over this region, soundings indicating as great a depth in the fiords as 385 fathoms being given on the chart. With these facts in view it is difficult not to believe that the present inland ice sheet once extended over all this portion of Greenland, passing out beyond the farthest limits of the present coast line into the open waters of Baffin's Bay. How far beyond the present limits of the coast line it passed, there are no facts yet known to justify a conclusion. Whether it met and coalesced with the ice sheet from the American side, seems idle speculation for the present. That it ever passed to the American side is disproved by evidence on that side, and that view is not now seriously entertained by any glacialist so far as known.

During its maximum extension the fiords must have been entirely filled and practically obliterated from the ice topography. As the ice sheet diminished in thickness, their influence became more marked till it became dominant in their region. It is probable that at one stage of the retreat, and that a very long one, confluent glaciers from Karajak, Itivdliarsuk, and other small fiords, united in one large Umanak

fiord glacier. During this stage the lower portions of the islands of Umanak and Ikerasak were covered and passed over by the ice, while the higher portions rose above its surface as nunataks. Probably well rounded when first exposed, frost action then and since has produced its results upon them as upon the sides of the fiord, and given them their present angular outlines as seen in Figures 22 and 23. That they must have been covered by the ice sheet is proved by the fact that the higher and neighboring summits of Nugsuak and of Storöen, and of the point northeast of Ikerasak are glaciated. It would be difficult to conceive of the glaciation of these summits by a sheet that did not cover all the lower summits. If this view is correct, then it follows that present angular characteristics such as are shown in these two cases of Umanak and Ikerasak do not preclude the possibility of previous glaciation, but only that its effects have been obliterated by later frost action and atmospheric erosion in general.

With this possibility in view the writer would suggest the same solution as possibly being applicable to the instances cited by Chamberlin[1] in the case of Dalrymple rock, and others farther north. This is merely a suggestion made with diffidence, as that region has not been visited by the writer. Indeed, the view of Dalrymple Island as given by Chamberlin presents a marked stoss and lee side, apparently in their appropriate positions as related to the mainland topography seen in the distance. (See Figure 25.)

Disko island, the largest on the coast of Greenland, is essentially a plateau having an elevation of 2,000 to 4,000 feet above the sea, with peaks rising to 5,000 and nearly 6,000 feet. It is surrounded on all sides by precipitous cliffs, which at the south just back of Godhavn rise vertically from not far above sea level to an altitude of over 2,000 feet. The base of the island is the same characteristic gneiss found in the Umanak region. This gneiss forms the whole of the low promontory, probably not exceeding 200 feet in its highest elevation, upon which is situated the settlement of Godhavn and in which is inclosed the fine little harbor of that place. This promontory is well glaciated by ice which had a westward motion, as noted by Chamberlin, and which the writer would have no doubt came from the mainland, as suggested by the above authority. The upper portion of the island consists of a thick cap of basalt resting upon the gneiss together with

[1] Journal of Geology, 2, 1894, p. 661.

FIG. 23. THE PEAK OF IKERASAK, SHOWING ANGULARITY, IN CONTRAST WITH THE ROUNDED OUTLINES OF THE LOWER AND MAIN PORTION OF THE ISLAND. IT IS BELIEVED THAT THIS PEAK HAS BEEN BURIED BY THE FORMER EXTENSION OF THE ICE. THE FOREGROUND IS OCCUPIED BY AN ESKIMO CHURCH.

FIG. 25. DALRYMPLE ROCK, AFTER CHAMBERLIN.

beds of sandstone which occur farther north, and especially along the Waigat. So far as known, gneiss does not rise far above sea level at any point on the island. Bowlders of gneiss, however, are scattered over the surface of the low promontory, and also in the steep, sharp valleys cut into the basalt cliffs and even upon the surface of the plateau. Opportunity was not given to search for evidence of the former extension of the inland ice of Greenland over the plateau surface of Disko, but if it did cover the Umanak fiord region and the Nugsuak peninsula, then it must also have enwrapped the whole of Disko Island, and have filled the Waigat. The bowlders of gneiss on the Disko plateau have probably come from the mainland, and do not, as suggested by Chamberlin, indicate a central core of gneiss rising to the surface in the interior of Disko, and from which they have been brought by the action of the present local ice-cap. So far as known to the present writer, however, there are no facts as yet observed on Disko to sustain either view. The view here presented simply follows directly in the line of the argument in the preceding pages.

On the American side of Davis Strait and Baffin's Bay the evidence of former glaciation is generally of the same character as on Greenland. The coast was closely skirted from the Strait of Belle Isle to north of Cumberland Sound, and landings were made at Turnavik in Labrador, on Big Savage Island, and the mainland of Meta Incognita in Hudson Strait, and on the south shore of Cumberland Sound at Niantilik harbor.

At Turnavik the rock is completely rounded into roches moutonnées, which near sea level retain a good degree of polish and have the striæ well preserved. Higher above the sea the rock has been more fully decomposed, and the striæ and polish have largely or entirely disappeared, though still seen on the highest summit, about 300 feet in altitude. The general direction of the motion here was about northeast, or about normal to the coast line. Erratics are rare, if not entirely wanting. The principal rock is a coarsely porphyritic granitoid gneiss cut by numerous dikes, one series of which is closely parallel in direction to the motion of the ice. Several dikes of this series have been eroded by the ice to a depth in some cases of 15 to 20 feet below the inclosing walls, as shown in Figures 26 and 27. In this, evidence is given of a large amount of preglacial decomposition in the dike rock as already noted by Professor Tarr. It also indicates a difference in conditions as compared with those of Greenland, where the

dikes are not eroded below the level of the inclosing rock. In the latter case the preglacial decomposition may not have been so great as in the former, due to a much earlier formation of an ice sheet in Greenland than in Labrador, or the erosive action of the ice has been greater, wearing down the whole surface below the limit of greater decomposition in the dike rock.

The coastal region of Labrador from Turnavik to Cape Chidley, as seen from the vessel, has the general appearance of a well glaciated region in its rounded contours and flowing outlines. In the vicinity of Cape Mugford, latitude 58° 30′, Table Hill, White Bear Island, and the Kig-la-pait mountains, 2,000 feet high, all appearance of rounding is lost when seen directly off shore, only sharp peaks or serrated edges being seen, giving the impression that the country here has never been overridden by an ice sheet. However, when seen from farther south in a direction transverse to the direction of the ice motion if it passed from the land toward the sea, as the evidence at the points actually visited proves, rounded outlines are sufficiently distinct to justify the conclusion that the ice did override this as well as other portions of the Labrador coast.

On the southern shore of Baffin Land along Hudson Strait the general rounding effects of glacial action are very apparent, but except near the sea level, polish and striæ have entirely disappeared. The rock is a very garnetiferous gneiss, sometimes highly ferruginous. Post-glacial decomposition has acted rapidly upon this rock, so that the entire surface is coated with its debris. Transported bowlders are also numerous, among them being many fragments of a finely crystalline limestone. Topographically the surface consists of a series of alternate high, narrow ridges, and deep, narrow valleys running about parallel with the strike of the gneiss, N-20-30-E (mag.). These valleys are possibly due to differential erosion of more highly calcareous layers. A few transverse valleys are apparently due to highly crushed and jointed areas. The ridges and hills are well rounded, the motion of the ice having been diagonal to the ridges. Professor Tarr[1] has already called attention to the fact that the principal topographical features here are preglacial, and to the evidence of the slight amount of glacial erosion here as compared with that farther south. It is also much less than any seen in Greenland.

[1] Am. Geol., 19, 1897, 194.

FIG. 26. WALL OF A DIKE AT TURNAVIK, LABRADOR, WHICH HAS BEEN REMOVED BY GLACIAL ACTION, INDICATING PREGLACIAL DECOMPOSITION.

FIG. 27. DIKE ERODED BY GLACIAL ACTION, INDICATING PREGLACIAL DECOMPOSITION. TURNAVIK, LABRADOR. (*From American Geologist.*)

At Big Savage Island the highest elevation visited is about 250 feet above the sea. Along this summit there is a well defined moraine continuing for several hundred feet with an average width of about 150 feet. It consists of fragments varying in size from pebbles to those 5 feet in diameter, nearly all well rounded. The general contour of the island is well rounded, showing the motion of the ice to have been southeasterly, but no striæ were found. All the valleys show a series of well preserved raised beaches, the highest being at an elevation of 250 feet above sea. These have been well described by Mr. T. L. Watson,[1] of the Cornell party. The rock here is a well banded gneiss, containing no garnets. The erratics are the same as the underlying gneiss, except a few limestone fragments and some dike rock. No fragments of the garnetiferous gneiss of the mainland were found, the motion of the drift having been sufficiently eastward to carry it north of this island. The limestone occurring in bowlders was not found in place.

At Niantilik harbor on the south shore of Cumberland Sound the gneiss is very highly disintegrated on the surface, so that at first sight there is an appearance of a non-glaciated surface, but rounded contours prevail and transported fragments are numerous. The direction of the ice motion here was southeasterly, nearly parallel to the axis of the Sound.

As previously mentioned, post-glacial decomposition is much more marked in Labrador, Meta Incognita, and Baffin Land, than any observed in Greenland, though, as noted by Professor Tarr in the article above cited, there are some indications of the comparatively recent withdrawal of the ice in these regions; still it was probably long anterior to the withdrawal of the Greenland ice-cap from the summits, 3,000 feet in altitude, or more, in the vicinity of Karajak and Itivdliarsuk fiords. On the latter the striæ and polish even are often still well preserved where the range of temperature is fully as low as in Baffin Land, and with greater variation between that of summer and winter, giving at least fully as good conditions for rapid disintegration.

In conclusion, the brief observations made along the Labrador coast and in Baffin Land would indicate that this whole country was once buried by a sheet of ice that in general had a motion outward toward the waters of Davis Strait and Baffin's Bay, and that this sheet

[1] Journal of Geology, 5, 1897, 17–34.

retreated from the immediate coastal region at least at an earlier date than did the Greenland sheet from the region of Umanak fiord. Though not proved by direct observation that it covered the highest peaks, yet by what has been seen it seems evident that, could these peaks be visited, they would furnish evidence to that effect. But as said in regard to the Greenland ice sheet, to determine how far the Labrador and Baffin Land ice extended into Davis Strait and Baffin's Bay, or whether it ever coalesced with the sheet from Greenland, there is as yet no evidence. Further investigation is needed of this most interesting problem.

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A NOTE ON SOME OF THE REQUIREMENTS FOR A SANITARY MILK-SUPPLY.

By WILLIAM T. SEDGWICK, Ph.D.

Received May 10, 1897.

It is now generally recognized that the milk-supply problem is one of the most pressing in American sanitation, and I am frequently asked to give an opinion as to the merits of this or that remedial measure. I have therefore thought it worth while to lay down very briefly, but I hope clearly, the fundamental principles which must be carefully kept in mind in seeking to introduce sanitary reforms into this important industry.

The fundamental, indispensable, and all-controlling requirement of a sanitary milk-supply is that milk when consumed shall be as nearly *normal* as possible. Normal milk is milk as it flows from the mammary gland of a normal animal, and a normal animal is obviously one that is healthy and well fed. From such an animal under normal conditions the milk supply of its young passes almost instantaneously, and without exposure to dust and air, from the milk ducts of the mother to the stomach of the suckling. Such milk is absolutely fresh, warm, and free from dirt. It is not only undecomposed, but nearly or quite free from the germs (bacteria) of decomposition.

Ordinary city milk, on the contrary, is neither fresh, warm, nor free from dirt, and if not already far on the road towards decomposition, is always richly seeded with bacteria. It is not always derived from healthy or well-fed animals, and is seldom drawn under clean and sanitary conditions, so that even at the outset it may be, and often is, very far from normal. It is also too often transported over long distances so that it still further loses its original freshness, and it is frequently manipulated by unclean, and sometimes by diseased, workmen. By the time it reaches the consumer, therefore, it is not only no longer normal milk, but usually stale, dirty, more or less decomposed, and sometimes also diseased.

Some of the steps to be taken in securing a more sanitary supply are easily deduced from the foregoing facts, and are as follows:

1. Milch cows should be healthy, well fed, well kept, and well cared for.

2. Milk should be derived from such cows only, and with all possible precautions in regard to sanitation and cleanness. Cows as sources of food should be tended as carefully as, or more carefully than, horses used only for burden or pleasure. The operation of milking should be looked after with special care. Above all, the hands of the milker should be carefully washed just before he begins to milk, his own personal cleanliness being even more important than that of the cow.

3. When drawn, milk should be immediately filtered and chilled, as means of retarding decomposition, and all articles with which it comes in contact, such as filters, pails, cans, etc., should be scrupulously clean.

4. The milk thus prepared should be delivered, if possible, at once.

5. If it is impossible to deliver the milk immediately, it will be impossible to deliver normal milk, and such milk cannot, in fact, generally be obtained in cities. The best that can be done, probably, is to deliver as speedily as possible two kinds of milk, viz.:

(*a*) The milk thus far described, kept as nearly normal as the conditions will allow.

(*b*) The same milk carefully *pasteurized* either (and preferably) on the farm where it is produced, or at some central point accessible from a number of farms, or, if this be impracticable, at some good distributing point in or near the city to be served.

The former (*a*), which may be called "chilled milk" or "raw milk" or "ordinary milk," will be preferred by some. The latter (*b*), which may be called "pasteurized milk" or "sanitary milk," will be preferred by many as being certainly free from the germs of infectious disease.

There can be no doubt that any individual or company which honestly strives to displace the present highly objectionable milk-supply of American cities by a supply such as has been here described, deserves, and will secure, the support and the confidence of the more intelligent portion of the community.

Biological Laboratory,
Massachusetts Institute of Technology.
May, 1897.

A FLAVOR-PRODUCING MICROCOCCUS OF BUTTER.

By SIMEON C. KEITH, Jr., S.B.

Received May 24, 1897.

In April, 1896, I was studying the effects of various bacteria upon cream, and in the course of my experiments I isolated from a mixture of bacteria growing in an agar tube a micrococcus that was found to produce a decided butter flavor and aroma when grown in milk or cream. This proved to be a new species, for which I propose the name *Micrococcus butyri-aromafaciens.*

It has always been the custom to allow cream to sour or "ripen" before churning it for butter, because after this process the butter comes better and more quickly, is of better texture and flavor, and keeps better than butter made from sweet cream. Lord Lister and Pasteur, many years ago, showed that the souring of milk and cream is due to a process of fermentation during which the milk sugar is converted into lactic acid, and that this is due to the activity of minute micro-organisms. It remained for Professor Vilhelm Storch, of Copenhagen, however, to introduce the use of pure cultures of milk-souring bacteria in butter making. Storch found that several kinds of acid-producing bacteria are concerned in the normal souring of cream, and he isolated three species that impart especially fine flavors to butter under favorable conditions.

A similar line of work was taken up by Professor Weigmann, of the Agricultural Experiment Station at Kiel, in Germany, and by Professor H. W. Conn, of Wesleyan University in this country.

Of the bacteria that have been described as producing a beneficial effect in the ripening of cream, *Micrococcus butyri-aromafaciens* (Figure 1) most nearly resembles Conn's *Bacillus No. 41*[1] (Figure 2), in its effects upon milk, but it differs in its morphological and in many of its physiological characters. It is a micrococcus growing at 37° and 20° C. It liquefies gelatin slowly, and does not grow well on potato. It may

[1] Storrs Agricultural Experiment Station, Bulletin 12, and Report for 1894.

be noted in this connection, however, that recent cultures on gelatin seem to show that the organism has lost to a considerable extent its power to liquefy gelatin during a year's cultivation in the laboratory.

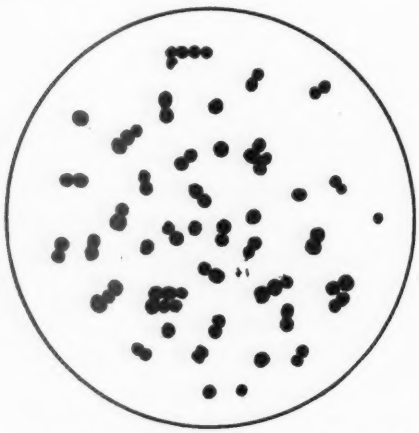

FIG. 1. MICROCOCCUS BUTYRI AROMAFACIENS.
MAGNIFIED 1,500 DIAMETERS.
DRAWN FROM A MICROSCOPICAL PREPARATION MADE FROM A GROWTH ON AGAR.

The culture of the micrococcus for use in creameries is propagated in bouillon in Fernbach flasks (broad flasks so constructed that a large surface of liquid is presented to the air). When ready for shipment, the culture is transferred to sterilized bottles under aseptic conditions and hermetically sealed by means of sterilized corks and melted paraffine. Put up in this way, the culture may be kept for an indefinite time without danger of infection by any other organism, but in the sealed bottles the micrococcus loses its vitality so rapidly that after eight days it will no longer produce the best results. Experiments made on a commercial scale show that cream ripened with the aid of fresh pure cultures of this organism produces generally better butter than the same cream ripened in the usual way. The distinguishing characters of the species are given in the following systematic description.

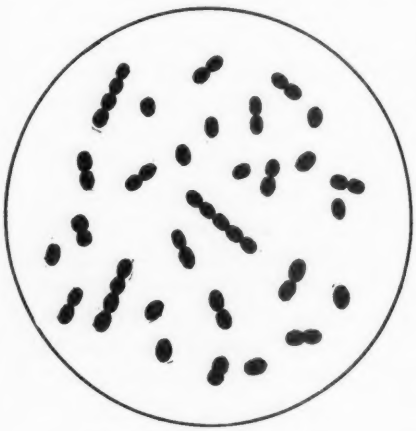

FIG. 2. BACILLUS 41, CONN.
MAGNIFIED ABOUT 1,500 DIAMETERS.
DRAWN FROM A PHOTOMICROGRAPH KINDLY SENT BY PROF. CONN.

MICROCOCCUS BUTYRI-AROMAFACIENS, Nov. Sp.

OCCURRENCE.	Isolated from a mixed culture growing on agar in April, 1896.
GENERAL CHARACTERS.	*Shape and arrangement:* A micrococcus occurring generally in pairs. *Size:* 0.5–0.7 μ in diameter, occasionally reaching 1μ. *Motility:* Non-motile. *Spore formation:* No spores. *Relation to temperature:* Grows rapidly at 37° and 20° C. *Relation to air:* Aërobic. *Relation to gelatin:* Slow liquifier. *Color:* Non-chromogenic (white). *Stain:* Stains well with carbol-fuchsin.
GELATIN.	*Stick culture:* Five days. The gelatin is liquefied in the form of a deep cup $\frac{3}{4}$″ in diameter. The liquefied gelatin remains clear, with a white film and sediment. The growth below the point of liquefaction is a moderately thick, white dotted line. *Plate culture:* Surface colonies: The colony first appears as a white raised dot which soon sinks in a pit of liquefied gelatin, and ultimately becomes surrounded by a slight whitish ring along edge of the liquefied gelatin. Submerged colonies: The submerged colonies occur as smooth spherical dots.
AGAR.	*Streak culture:* A very white, smooth, shining growth, which is fairly abundant. The growth is of equal thickness throughout. *Plate culture:* Characters of no diagnostic value. *Lactose-litmus-agar:* Litmus reddened slightly.
POTATO.	There is very little growth on potato. In two weeks it appears as a very thin, white line, barely visible.
MILK.	Not coagulated. A slightly sourish, pleasantly aromatic, "buttery" flavor. Slightly acid.
SMITH SOLUTION.	No gas produced. The growth occurs mostly in the open limb of the fermentation tube, the bouillon of the closed limb being only very faintly turbid.
NITRATE.	Reduced to nitrite. Recent cultures do not seem to give this reaction very strongly, although when first isolated it was very marked.
BOUILLON.	Two days, 25° C. Very cloudy with sediment. One week, no further change. Two days, 37° C. Very cloudy with sediment and ring growth on tube at surface of the liquid.

SOME FUNDAMENTAL PROPOSITIONS RELATING TO THE DESIGN OF FRAMEWORKS.[1]

By FRANK H. CILLEY, S.B.

Received November 27, 1896.

Part I. — The Evolution of the Framework — The Arbitrary Nature of the Distribution of Stress in Indeterminate Forms.

In ancient times the builder practically disposed of but two sorts of material suitable for his purposes — stone and wood. The former, adapted only to resisting compression, permitted of no higher development in construction than the arch; and the latter, although in addition well suited to the carrying of transverse loads, was so limited by its tendency to split and shear, and the consequent difficulty of making strong connections, that it hardly admitted of a more complex combination than the braced beam. These forms appear to have been known almost from time immemorial, and it is possible that to this day we should have remained limited to them had not metal, with its more extended qualities, come within the range of building materials. Metal, it is true, had also been known from most ancient times, but only as rare and costly material. It is the very modern cheapening in production of certain of its forms, particularly of iron and steel, which has permitted of its employment for ordinary structural purposes, and thereby given rise to the numerous forms of construction existing to-day.

These forms, however, did not at once come into existence, but rather were gradually evolved from the earlier forms in the attempt

[1] The principal results of this article, viz., the arbitrary nature of the distribution of stress in an indeterminate framework and the economic superiority of statically determined construction, were obtained in February, 1896, and embodied in a communication of that date to Professor George F. Swain. In June, 1896, Professor Swain returned the manuscript of the above communication, advised rewriting it in popular form, and suggested a second and independent proof of the economic superiority of statically determined frameworks. The present article, completed in July, 1896, was the outcome.

to utilize with higher economy the still very expensive new material, taking advantage of its resistance to tension as well as compression, and of the ease with which it could be given any desired form, and the strength with which different pieces could be united. Commencing with mere imitations of the manners of use of the older materials, it was soon observed that such massive construction in metal was as unnecessary as uneconomical, and sections such as the T, the I, the O, and others of great stiffness, with relatively little material, were developed and employed. With the extensive production of malleable metal came the construction of built-up sections; the plate girder sprang into existence, and was followed by the more economic lattice girder; then the latticing was gradually reduced and concentrated in fewer and fewer pieces, thus approaching more and more the typical framework of to-day, in which each piece is called on to perform but a single definite and simple function, the support of a direct stress of tension or compression.

Such is, briefly, the history of the evolution of the framework, at least for Europe. There it resulted simply from the reduction of the solid beam and arch to a skeleton form in the effort to secure economy. The new forms were regarded merely as modifications of the old, and the recognition of the existence in them of a new type of construction was and is far from universal.

In America the history of this evolution runs somewhat differently. Here wood was extensively employed in early times, and we were already thoroughly familiar with the idea of bracing a timber construction with stiff struts when metal for structural purposes became a possibility with us. The value of its employment for tension pieces was at once appreciated; new forms in which it could thus be utilized were rapidly devised, and the framework of mixed metal and wood, or all metal, soon became a distinct type of construction. But evolved, as it had been, in so different a way, it very naturally differed in many important respects from the European production; and our manner of viewing it was also different and more correct, yet not wholly free from bias, for in many cases the framework had developed as bracing or stiffening for certain principal members, and still continued to be regarded only in that light.

In the process of evolution of structures here outlined theory has played a small, although ever-increasing, part. The arch and beam in the far past were dimensioned almost wholly from experience,

with the aid, perhaps, of a little most general and loose reasoning. The later structures have hardly followed a more royal road. It was largely through experience and observation that their proportions were at first determined, certain sources of weakness having been noted in similar existing structures, which it was sought to remedy through larger dimensions or the introduction of new parts. Approximate theories based on the comparison with beams were employed at first, in Europe; hardly less arbitrary *views* as to how the stress was shared were early in vogue here. The fact is, the structures themselves were in fault, being of such complexity, from an analytic point of view, as to make their exact solution practically impossible even by eminent mathematicians of this late day. Moreover, the early forms on which they were based, the beam and the arch, with all their apparent simplicity, offer analytic problems of the greatest difficulty which have not yet received exact solutions.

However, as the structures became more and more complex outwardly they had been growing more and more definite in the functions of their parts; each part was there for a more and more definite purpose, and only for that; the theory was in reality growing simpler. And when at last came the idea of so constructing that each part should have but a single definite function — to resist a push or a pull — the theory was brought within the range of application, the idea of the typical framework was conceived, and it became possible for theory to take the reins and drive where before it could not even follow.

All the older forms of construction were characterized by a complex interdependence of their inner stresses and strains. A mere knowledge of the form of such a structure did not suffice for a determination of the inner stresses due to a given loading. The elastic nature of the material and its variations had to be considered; the theory of elasticity, as well as the elementary principles of statics, was involved, and in a most complex manner; the structures were highly statically indeterminate. But in the development of structures this degree of indetermination became less and less, while what then remained became more and more simply and sharply definable; and in the typical framework we can define and determine all the stresses through simple linear equations without aid of the calculus. Such a theoretic construction may even be wholly free from indetermination, that is, statically determined.

The theory of the typical framework, then, may be said to be, not only possible, but relatively simple. It should therefore be clearly understood by engineers and others who employ such construction. They should know just what powers of design lie in their hands and how they may best use them. Neither prejudice due to older empirical ideas, nor illusions based on imperfect knowledge, should be able to lead them astray. It is to contribute to this end that the present article was written, to make somewhat clearer the general conception of the nature of these most important structures, and to point out sharply some very important fundamental propositions concerning their design commonly overlooked or unknown.

The leading characteristic of the typical framework is, as has been pointed out, the simplicity in function of its parts. Each member or bar should be called on to support only pure tension or compression. Such is the ideal condition for which engineers and constructors should strive; for through its attainment alone the highest degree of certainty in the calculations, and of efficiency in the employment of material, is possible. In order to attain this end, however, the framework must be constructed in a certain manner. Its rigid members (or bars) must be connected with each other only at their ends and by joints whose resistance to slight changes in the relative angles of the bars is negligible; and all external forces, such as loads and reactions, must be applied practically at the joint points. The bars are preferably of prismatic form, and their axes should meet at a point in each joint. Such, in brief, are the most essential requirements in the details. As to the general form, *that* may be defined simply through the geometric figure of lines formed by the axes of the bars. In order that our framework should be capable of performing its function of properly supporting loads it is necessary that its figure should be defined so fully under the given reaction limitations, through the lengths of its lines, that no change in its form would be possible without corresponding change in some of these lengths. The knowledge of the figure alone suffices to determine the static conditions which the stresses in such a framework must fulfill.

This description of the typical framework is more or less ideal. The degree of freedom at the joints of actual structures is far from what it should be. The loads are often applied away from the joints where they should not be, and in any case, our bars being material and of more or less considerable dimensions and weight, transverse

loading from wind and weight is not wholly avoidable. Thus the absence of other than direct stresses in its members is an ideal condition for a framework, which in practice is never more than partially attained. Nevertheless, here, for the sake of simplicity and to elucidate some characteristics of frameworks, otherwise easily masked by the complications of a more exact study, we will confine ourselves chiefly to this ideal case.

At each joint of an ideal framework we have to deal only with forces meeting at a point, the joint center. These forces are partly known (loads) and partly unknown (bar-stresses and reactions at supports), but they must at each joint satisfy the static conditions of equilibrium of forces at a point — that the algebraic sum of the components of these forces (inner and outer) in any three directions (non-planar framework) shall be zero.

This furnishes us with three equations between the known forces (loads) and the unknown forces (bar-stresses and reactions) for each joint, and, if the framework have n joints, this gives $3n$ such conditions in all, which are statical requirements following purely from the figure of the framework, and which must in any case be fulfilled. Let m be the number of bars and r the number of limitations at the supports of the frame, then we have $m + r$ unknown quantities to determine for each loading of the framework. These may not be less in number than our $3n$ static equations if our framework is stable (of determinate figure). Should they equal in number the static equations ($m + r = 3n$) the framework may be calculable without further data, *e. g.*, statically determined. But should they be in excess of the number of static equations ($m + r > 3n$), then other "elastic" equations will be necessary to complete the determination; that is, our framework will be statically indeterminate. It is to this latter general class of frameworks, of which the statically determined may be regarded as a limit form, that we will now turn our attention.

In statically indeterminate frameworks, then, we have more unknown quantities than equations furnished by statics. What follows? Usually it is answered, certain further conditions furnished by the theory of elasticity enable us to complete the solution. But this is not precisely the truth.

If only the sections of the bars (or, in the case of a new design, the intensities of stress to exist in them) be given in addition to the geometrical figure of the framework, and its loading, as usually

is, in fact, the case, the problem remains indeterminate. It is only when both the sections *and* the intensities of stress are given for all the superfluous bars and the sections or intensities of stress of the non-superfluous bars, or, what is equivalent, the sections and the *exact* lengths, center to center of joints, of the unstrained bars are given, that the problem becomes determinate. The last of these cases is the one on which calculations are very frequently founded, yet rather insecurely, as will later be seen. If it is a case of new design, we will ordinarily have given only the allowable intensities of stress in addition to the figure of the framework and its loading. Then, as will now be shown, we can in an infinite variety of ways fix the stresses in the bars or determine their sections.

First consider the determination of the stresses in this latter case. In our given data there is nothing which limits the stresses further than the $3n$ equations of statics already noted, and which are absolute. So, if we have k unknowns in excess of these, we have a k^{tuple} degree of indetermination, which admits of ∞^k different solutions, as the principles of analysis show. Or, to make this more objective, we find that the stresses in as many bars as are in excess may freely be assumed, these bars being any such that the remaining bars form a statically determined framework for the support of the given loading, together with these assumed stresses regarded as outer forces. Our procedure in calculation is simply to regard these superfluous bars as removed and replaced by pairs of equal and opposite forces, acting at their end joints and equivalent to their assumed stresses. Thus the calculation of the stresses in the remaining bars reduces to that for a statically determined framework, and may be proceeded with by any of the usual methods. From the stresses thus found, and the given intensities of stress, the sectional areas at once follow. It only remains to insure that these given intensities be attained. This we may do, at least in theory, by giving to the superfluous bars suitable primary lengths.

In such a design it is to be noted that primary stresses, that is, stresses without loading, would exist.

Suppose that instead of assuming the *stresses* in the superfluous bars we had assumed their *sections*. Thence from the given intensities of stress would at once follow their stresses — thence, as before, the stresses in the remaining bars and their sections.

We must observe, however, that in no case could we assume

more of the stresses than there were superfluous bars, the requirements of statics not permitting of that. And should we assume the sections of more than the superfluous bars, the intensities of stress in a corresponding number of bars could (in general) no longer be as given.

Another way of viewing our power of varying the details of a design of a framework of given figure, loading, and intensities of stress is found in the following. Under these conditions we may design so that the stresses shall be apportioned among the various bars in any way consistent with the static conditions of equilibrium of forces.

In all these cases the intensity of stress borne by each bar may be anything we please (within limits). It may therefore be set at the maximum allowable value for each bar, so that our infinite variety of proportions may all be made to fulfill the requirement, that each bar shall be carrying its maximum allowable load, that is, working at its maximum efficiency. And in this connection be it noted that the maximum allowable stress may be set independently and, if desired, differently for each bar; so that, if bars in certain positions should be more or less favored than bars in other positions, or if bars in different positions be of different materials, as wire cable for tension members and cast metal for compression members, or even of entirely different substances, as iron or rope tension members, and stone or wood compression members, we can employ the corresponding working stresses and still retain our powers of variation. That is, these powers hold perfectly for mixed wood and iron truss construction, or for suspension bridges, whose cables and numerous ties are of wire cable, and the stiffening truss of other material.

Illustrations of Arbitrary Proportioning of a Statically Indeterminate Framework.

The principles of design expounded in what precedes will be rendered clearer by consideration of the following simple illustrations:

Let it be required to design a square frame ten feet on a side, with two diagonals, supported by a vertical wall and supporting a load of 100 tons at its lower outer corner (see Figure 1). We are required to employ a maximum intensity of stress of, say, five tons per square inch in tension, four tons per square inch in compression

(without regard to post formulæ, for simplicity). The material has uniformly a modulus of elasticity of, say, 14,000 tons per square inch.

Let us regard the diagonal from the lower inner corner to the upper outer corner as the superfluous bar, and we will require it to take, say, 40 tons compression. Consider this bar removed and replaced by a pair of forces of 40 tons. Our frame thus reduces to

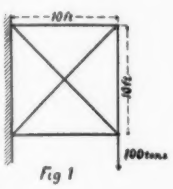

Fig 1

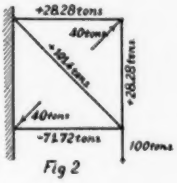

Fig 2

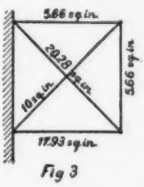

Fig 3

a statically determined frame, whose stresses are shown in Figure 2. The section areas at once follow from the prescribed intensities of stress, as shown in Figure 3.

It only remains to calculate the primary lengths of our bars which shall insure the assumed distribution of stress. The primary lengths of all except the superfluous bars are made simply the dimensions of the figure of the frame in the unstrained state, as in Figure 4. But the primary length of the superfluous bar can only be calculated

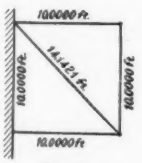

Fig 4

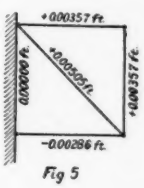

Fig 5

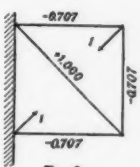

Fig 6

from a knowledge of its length when the frame is distorted by the stresses shown in Figure 2. There, the intensities of stress being five tons for members under tension and four tons for members under compression, tension bars were lengthened $\frac{5}{14.000}$, or .000357 of their lengths, and compression bars were shortened $\frac{4}{14.000}$, or .000286 of their lengths, the consequent strains being as shown in Figure 5. Note that no strain along the wall is assumed. The shortening of the missing diagonal corresponding to these strains we find most simply by application of the method based on the principle of virtual work. Let us imagine a pair of forces of unity each, applied at the

end joints of the missing diagonal, then the stresses it would cause are shown in Figure 6; and if we multiply each of these with the corresponding strain shown in Figure 5 and sum these products algebraically, the result will be the work of unity in shortening the distance along the missing diagonal. This we find to be—

$$
\begin{aligned}
(-\ .707) \times (+.00357) &= \quad -.00252+ \\
(-\ .707) \times (+.00357) &= \quad -.00252+ \\
(+1.000) \times (+.00505) &= +.00505 \\
(-\ .707) \times (-.00286) &= +.00202 \\
&\overline{\qquad\qquad} \\
+.00707 - .00505 &= +.00202 \text{ ft. units.}
\end{aligned}
$$

The missing diagonal was therefore 14.1421 — .0020 = 14.1401 feet long. But this length corresponds to a state of compression of 4 tons per square inch. The unstrained length would therefore be $\frac{4}{14.000} \times$ 14.14 feet, or .00404 feet greater, that is, 14.1401 + .0040, or 14.1441 feet, which differs from the length of the other diagonal bar .0020 foot.

To find what primary stresses this design involves we proceed as follows: Our superfluous bar is designed .0020 foot longer than it would be were there no primary stress. It must therefore be under such compression, and the rest of the frame under such consequent stress, that the shortening of our diagonal and the separation of the joint centers at its ends together amount to this .0020 foot. Now one ton *tension* in our diagonal would cause the stresses shown in Figure 6, and the strains corresponding are as in Figure 6*a*. The shortening of the diagonal distance calculated as before is—

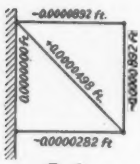

Fig 6a

$$
\begin{aligned}
(-\ .707) \times (-.0000892) &= +.0000631 \\
(-\ .707) \times (-.0000892) &= +.0000631 \\
(+1.000) \times (+.0000498) &= +.0000498 \\
(-\ .707) \times (-.0000282) &= +.0000199 \\
&\overline{\qquad\qquad} \\
&= +.0001959 \text{ ft.,}
\end{aligned}
$$

therefore for every ton primary *compression* in the superfluous diagonal its end joints in the frame will *separate* .000196 foot. But the diagonal itself, whose section is 10 square inches and length 14.1441 feet, would be shortened $\frac{1 \times 14.1441}{10 \times 14{,}000} = .000101$ foot by every ton

of compression it bears. Every ton of compression in the diagonal therefore corrects (.000196 + .000101) foot = .000297 foot of the difference of its length and the length for no primary stress. The whole difference, or .00202 foot, will be corrected by $\frac{.00202}{.000297}$ tons, or 6.8 tons, which is the primary stress in the diagonal. The complete distribution of primary stress is shown in Figure 7.

It is interesting to note with what extreme exactitude the lengths of the bars must be calculated, and how accurately the construction would have to be carried out. Moreover, the assumption that the distance along the wall between the points of support remains invariable, although similar to those usually made in calculation of statically indeterminate structures, is evidently open to objection. Any curving of the posts under compression would cause further error,

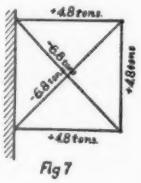

Fig 7

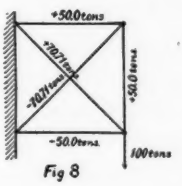

Fig 8

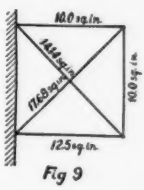

Fig 9

the bar axis shortening from $2d^2$ to $2.7d^2$, where d is the ratio of middle deflection to length. $d = \frac{1}{200}$ corresponds to an error of .7 to .9 ton per square inch.

To illustrate another apportionment of stress than the arbitary designation of the stress in the superfluous bar, let us take a framework of the same figure as before and so design it that the two diagonals shall equally share the shear. Then our stresses at once work out as in Figure 8; and with the same allowable intensities of stress as before, that is, five tons per square inch in tension and four tons per square inch in compression, the section areas become as shown in Figure 9. As for the primary *lengths* in this case, since the stresses are all of the same intensities *and same kind* as in the preceding case, these remain unchanged; but the relative areas are no longer the same, so that the primary *stresses* necessary to account for the .0020 foot excess of length of the superfluous diagonal will presumably not be as before. We find for this case, in Figure 6*b*, the strains corresponding to one ton *tension*

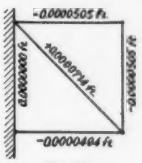

Fig 6b

in this diagonal (see stresses in Figure 6). The shortening of the diagonal distance, calculated as before, is—

$$\begin{aligned}(-.707) \times (-.0000505) &= +.0000357\\ (-.707) \times (-.0000505) &= +.0000357\\ (+1.000) \times (+.0000714) &= +.0000714\\ (-.707) \times (-.0000404) &= +.0000286\\ &= +.0001714 \text{ ft.},\end{aligned}$$

for every ton tension in the superfluous diagonal, or the same increase for every ton compression. The diagonal itself would be shortened $\frac{1 \times 14{,}1441}{17.68 \times 14{,}000}$ or .000057 foot for each ton compression. Its primary stress must therefore be given by $\frac{.00202}{.000171 + .000057} = \frac{.00202}{.000228}$ = 8.8 tons. The complete distribution of primary stress is shown in Figure 10.

As a final case in design, which only too frequently is met in practice, consider that in which the exact primary lengths of all bars are given in advance. Usually in this case these lengths are given by the requirement that there shall be no primary stresses; but then, as previously noted, under loading as many intensities of stress as there are superfluous bars cannot be set in advance, but will be determined through the intensities assumed to exist under loading in, say, the now superfluous bars. The stresses or sections of our superfluous bars, however, remain, as before, wholly within our control. Let this be illustrated with the same framework as before, and suppose the same distribution of stresses as in Figure 8 assumed. The intensities of stress in the now superfluous bars being the same as before, we know the superfluous diagonal will have been shortened .0020 foot, which corresponds to an intensity of stress of $\frac{.00202 \times 14{,}000}{14.14}$ = 2 tons per square inch; therefore, since this bar carries 70.7 tons, its section area must be made $\frac{70.7}{2} = 35.35$ square inches. All other sections remain as in Figure 9. This gives our superfluous bar double the area and volume it had in the preceding illustration, showing clearly the loss of efficiency which may result from insisting on the condition of no primary stress in the framework.

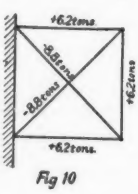

Fig 10

In this, as in the preceding two illustrations, we could have assumed

the section areas instead of the stresses in the superfluous bars; but should we assume the section areas of all the bars, as well as a condition of no primary stress, then all stresses and stress intensities would thereby be determined and no longer within our control. And such is, perhaps, just what most frequently occurs with present methods of design as well as in the calculation of existing structures. Usually in design all the sections are tentatively assumed (in an existing structure they are given), and the supposition of no primary stress being made, often unwittingly, the problem becomes wholly determinate. In an existing structure the supposition of no primary stress will be more than questionable, and in a design it will be a foolish limitation, for the intensities of stress should be as prescribed, and should determine the design — not be determined by it.

To illustrate the results and one method of calculation for this case, suppose we were given the framework, with sections as in Figure 9, but subject to no primary stresses. Regard the same

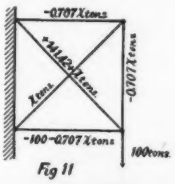

Fig 11

Fig 12

diagonal, as before, as superfluous, and let x be its stress; then the stresses in our frame would be as shown in Figure 11. The consequent changes in lengths of bars we find, as usual, by multiplying these stresses by the bar lengths and dividing by the areas and modulus of elasticity, the result being as shown in Figure 12. Now the shortening of the line of the superfluous diagonal calculated from the strains of the other bars by aid of Figure 6, as before, is —

$$\begin{aligned}
(-.707) \times (-.0000505x) &= +.0000357x \\
(-.707) \times (-.0000505x) &= +.0000357x \\
(+1.000) \times (+.01010 + .0000714x) &= +.0000714x + .01010 \\
(-.707) \times (-.00571 - .0000404x) &= +.0000286x + .00404 \\
&= +.0001714x + .01414
\end{aligned}$$

$= -.0000571x$ (see Figure 12), the *shortening* of the diagonal itself. Thus $x = \dfrac{-.01414}{.000171 + .000057} = \dfrac{-.01414}{.000228} = -62.0$ tons, and the

actual distribution of stress under these circumstances would be as shown in Figure 13. There is by no means an equal division of shear between the two diagonals. As for the intensities of stress, they are as shown in Figure 14. It is seen that the tension diagonal and the lower horizontal are both overstrained, and the remaining bars bear less than their fair loads if we keep to the earlier criterion of 5 tons allowable for tension and 4 tons for compression. The results are some 12 per cent out. If this were a design it

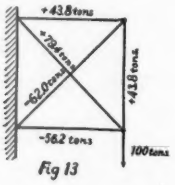

Fig 13

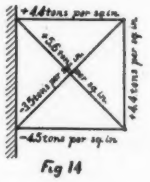

Fig 14

might be attempted to improve these results by correcting the sections, but with little satisfaction so long as primary stresses were excluded. The system is, in fact, one of design by blind trial, and well illustrates the general lack of acquaintance by designers with what are really very elementary principles of the distribution of stress in frameworks.

Part II. — Economic Considerations — Inferiority of Indeterminate Frameworks.

At first it would seem as if our power of varying the distribution of stresses and so distribution of material in our framework, as set forth in the preceding part, was almost unlimited, but a little closer observation will show that in reality it only extends to as many options as there are superfluous bars, thenceforth all being fixed and determinate. Moreover it must be noted that the power of variation only extends over such parts of the frame as are affected by the stresses in the superfluous bars, that is, the statically indeterminate portions of the framework.

Further, in order to make use of even this power with which the designer is invested, exceedingly exact and laborious calculation of all elements in the design is *absolutely* essential, in particular it being necessary to calculate very exactly the lengths, center to center of joints, of the unstrained bars, and it is equally essential that the con-

struction should be exactly carried out in order that the supposed conditions of the calculation be fulfilled. How very exact work this means, will be appreciated from the fact that errors of a thousandth of a foot in 10-foot bars are far from negligible, involving errors in stress as great as 30 to 35 per cent., the allowable stress. Moreover it is seen that the calculation should properly take into account the yielding, both permanent and elastic, at the reactions, and at the joints and splices. And aside from the excessive accuracy necessary, the (often considerable) primary stresses we may find it desirable to introduce may create difficulties. In the face of these and other real and very important objections to the employment of statically indeterminate frameworks, objections from which statically determined frameworks are wholly free, it may well be asked why should indeterminate frameworks be employed; what real advantages do they possess?

It has already been pointed out how the evolution of the framework was an outcome of experience, rather than theory. And experience is a very slow teacher. But many will follow in her tracks alone, paying little heed to the straight path to their goal which theory points out to them. Thus the early frameworks having been statically indeterminate in their figures, as well as departing from the ideal in other respects, such as in the fixtures of the bars at intermediate points as well as at the ends, and the use of stiff joints, all these defects have been perpetuated even in modern constructions. And the constructor, to whom these features are pointed out as defects, is apt to question the theory on which the criticism is based. He may even have a theory of his own which proves to his satisfaction the superiority of these very features. And as the theory of such structures is exceedingly difficult and complicated — so much so that closely approximate single solutions have only been attained by the most modern methods of calculation, and these in but very few instances — the difficulty of correcting these delusions by an analysis that is both general and exact may be imagined. Only for the typical framework are we in a position to make such an analysis which shall be fairly general, and at the same time even approaching exactitude. This we will now undertake.

Three of the most important claims urged in favor of frameworks with superfluous bars are that they are stiffer, that they are safer, and that they are more economic in material used. Let us examine these claims.

The deflections of a frame under a given loading are absolutely determined by the intensities of strain in the bars of the statically determined portion left after eliminating the superfluous bars. If the bars of this portion have a determined maximum stress, therefore strain under load, the deflections are also determined, whatever the number of superfluous bars. Only by a reduction in the intensities of strains, that is, in the efficiencies, can increased stiffness be secured, and this is independent of the use of superfluous bars. The first claim, therefore, is unfounded for a fair basis of comparison, fixed intensities of strain.

The rupture of a bar of a framework with superfluous bars will not less surely cause its downfall than in the case of a framework every bar of which is necessary, unless the ruptured bar chances to be one which may be regarded as superfluous. The ratio of such bars to the whole gives a rude measure of the extent of this possible security. But its true value depends on whether such a bar being ruptured, the rest of the bars are strong enough to bear the load alone, especially when we consider the probable accompanying shock of rupture. A certain degree of additional security undeniably exists, but what?

There remains the claim of superior economy. But before making a more exact examination of this, let us ask on what is such a claim based. And to which of the innumerable designs of frameworks of a given figure supporting a given loading, which have been shown in Part I of this paper to exist, does it apply? We may answer that the claim is based on practically nothing except the fact that the continuous girder and the arch are more economic than simple girder spans. And this unfair comparison may now be set aright by the introduction of the really comparable statically determined forms, the cantilever and the three-hinged arch. As to the second of the above questions, the partisans of frameworks with superfluous bars, being for the most part unaware of these possible variations, could give no answer. We will, then, answer for them. A design in which the stresses of as many bars as are superfluous are zero, that is, a statically determined framework whose figure is included in the given figure, will be the most economic. This, a direct contradiction in terms, of the claim of superior economy of an indeterminate form, we will now prove.

Suppose we have given any one of the possible designs by its bar-sections, and the stresses in the bars under the given loading. Let us determine the effect on the quantity of material employed, of varying

this distribution of stresses and material without varying the external loading or the intensity of stress in any bar. For the sake of simplicity we will suppose that the intensities of stress in question are simply the mean intensities obtained by dividing total stress by area of section. And we will suppose the mean strain to be proportional to the mean stress, that is, the shortening due to possible curvature of a bar to be negligible. The first assumption which neglects the effect of post formulæ on design we will correct later. The second assumption is usual in all calculations, and since the shortening of the axis due to bowing of a bar is between $2d^2$ and $2.7d^2$ of its length, d being the ratio of deflection to length, this supposes no deflection exceeding, say, 1/200 the length which would correspond to a shortening of from .00005 to .000067 the length, and with a modulus of 14,000 tons would be equivalent to the effect of an intensity of stress of from .7 ton to .9 ton per square inch. It may be objected that these and even larger deflections are both possible and probable, and that it is unfair to neglect them. This may be true, but since this supposition alone makes the calculation of frameworks with superfluous bars possible, it will at least not be objected to by the partisans of such construction.

We assume, then, our external forces invariable, our bars always straight, and the sections varying in the same proportion as we vary the stresses, so that the mean intensities of stress are maintained constant. Let our given frame have m bars, of which k are superfluous under the given reaction conditions. Let any such k of the bars as may be regarded as superfluous be selected out leaving us with a statically determined framework of $m - k$ bars which we will denote by the numbers 1 to $m - k$ inclusive. Consider in connection with this statically determined portion any one of the superfluous bars which we will denote as the bar $m - k + 1$. Now suppose under the given loading and the stresses in the remaining superfluous bars, maintained constant by the unvarying mean intensities of stress in our bars 1 to $m - k$, we vary the distribution of stress between these $m - k$ bars and the bar $m - k + 1$, varying the sections correspondingly, and note the effect on the quantity of material employed. Let e denote any one of the bars 1 to $m - k$, S_e its stress, l_e its length, and σ_e its (constant) mean intensity of stress. And let S'_e denote the stress (tension) that would be caused in the bar e by a tension of unity in the bar $m - k + 1$, all other superfluous bars being removed. Then any increase in tension dS_{m-k+1} in the bar $m - k + 1$, since the stresses

in the other $k - 1$ superfluous bars are kept constant, will simply be accompanied by the corresponding changes in stress $dS_e = S'_e\, dS_{m-k+1}$ in each bar e of the remaining bars. Their corresponding changes in section will be $dA_e = \frac{dS_e}{\sigma_e} = \frac{S'_e\, dS_{m-k+1}}{\sigma_e}$ and in volume will be $dV_e = l_e dA_e = \frac{l_e S'_e\, dS_{m-k+1}}{\sigma_e}$ or for the whole frame, summing over all $m - k + 1$ bars, and putting i for 1 to $m - k + 1$ and $S'_{m-k+1} = +1$.

$$dV = dS_{m-k+1} \sum_{1}^{i}{}^{m-k+1} \frac{l_i S'_i}{\sigma_i}$$

It is to be observed here that l_i is always $+$ and σ_i and S'_i either $+$ or $-$, according as they are tension or compression.

Now the expression for dV thus formed will in general be either $+$ or $-$, according to the sign we give dS_{m-k+1} (unless very exceptionally

$$\sum_{1}^{i}{}^{m-k+1} \frac{l_i S'_i}{\sigma_i}$$

be zero), so that by suitably choosing the sign of dS_{m-k+1} we can in general effect a saving in material through our variation in distribution of stress. But during this variation the quantities l'_i and S'_i remain unchanged whatever its range, and the σ_i can only change through the stress in some bar becoming zero, that is, through our $m - k + 1$ bars reducing to a statically determined combination of $m - k$ bars bearing the same outer loading and the same stresses of the remaining superfluous bars, and having the same intensities of stress as before. It follows, therefore, that we can so vary our distribution of stress as to eliminate one bar of our $m - k + 1$ bars, and leave the remaining bars performing the same function under the same conditions, but with the use of less material (exceptionally with the same quantity of material).

In the same way we may now treat these $m - k$ modified bars together with one of the remaining superfluous bars when we shall again find that we may eliminate a bar to advantage, and so, successively eliminating bar by bar, we eventually arrive at a statically determined

framework of $m - k$ bars in all, so modified in section that they bear alone the whole original loading with stresses of the same kinds and intensities as those in the corresponding bars of the given framework, and with the employment of less material (or by exception the same amount).

Thus, under the limitations of our analysis, it appears that

No given design of a framework with superfluous bars may support a given loading with more economy of material than some statically determined framework whose figure is included in the figure of the given framework, and whose stresses are of the same kinds and intensities as those in the corresponding bars of the given framework.

And since this is true of *any* design of a framework of a given figure and loading, it follows that (under our limitations)

The framework which will support a given loading most economically will always be statically determined.

Another very simple and much more objective proof of the same proposition is the following, subsequently suggested by Professor George F. Swain, to whom the author is indebted for his early training in the statics of structures, and also for a review of the present article and much valuable advice in connection with it. To Professor Swain's steadfast support of statically determined construction was due the author's interest in this subject, leading to the study whose outcome was the general demonstration of its economic superiority, here given, it is believed, for the first time. We may hope that this superiority, hitherto frequently claimed for indeterminate construction, will henceforth be recognized universally as an attribute of determinate construction.

Suppose we have given any framework with superfluous bars, and its loading, stresses, intensities of stress, etc., and that we have found a series of statically determined frameworks, whose figures are included in the figure of the given framework, all of the bars of the given framework being represented in these separate frameworks, and in each of which the given loading would cause the same kind of stress in the bars as in the corresponding bars of the given framework. It is evident, then, that if to each of these statically determined frameworks we assigned a suitable fraction of our given loading, the stress in any bar of the given framework would be equalled by the arithmetic sum of the stresses in the corresponding bars of the statically determined frameworks; and further employing the same mean inten-

sities of stress in the design of all corresponding bars of these statically determined frameworks and the given framework, the section area of any bar of the given framework would be equalled by the arithmetic sum of the section areas of the corresponding bars of the statically determined frameworks, since all stresses in all corresponding bars are, by supposition, of like kind. It follows that the total volume of material in the given intermediate framework is also equalled by the arithmetic sum of the volume of material of all our statically determined frameworks. Therefore, the efficiency of our given framework, measured, if we please, by the quotient of its total loading divided by its total volume of material, will lie between the efficiencies of the most and the least efficient of our statically determined frameworks, which may in a way be regarded as elements into which our given framework has been decomposed. It follows, therefore, since the given framework cannot be more efficient than the most efficient of these elements, that *the given framework cannot be more efficient than some statically determined framework, whose figure is included in the figure of the given framework, and the stresses in whose bars are like in kind and intensity to the stresses in the corresponding bars of the given framework.* (Q. E. D.)

It may occur that we are not able to find the necessary statically determined forms whose figures are included in the figure of the given framework, and the stresses in whose bars under the given loading are like in kind to those in the corresponding bars of the given framework. If there be k superfluous bars, we see that $k + 1$ such statically determined frameworks are necessary in order that each bar of the given framework should have a corresponding bar in at least one of these.

Suppose, now, that we could not find a statically determined framework under our conditions which should include a certain bar. It can readily be shown that we can at least always find some one statically determined framework fulfilling our conditions, and so, by supposition, not containing this bar. Then the introduction of this bar, with stress of the amount and kind it must have in the given framework, into this statically determined framework, clearly must simply increase the stresses (therefore correspondingly the section areas) of all bars of this framework which are affected at all thereby, and our bar forms with our statically determined framework a statically indeterminate

framework with one superfluous bar, whose stresses are like in sign and intensity those in the corresponding bars of the given framework, but which, supporting only the same loading as our first or primary statically determined framework, while containing more material, is less efficient than this primary. Thus, we can *always* find a series of statically determined frameworks and of indeterminate frameworks with one superfluous bar derived from these primaries, whose figures are included in the figure of the given framework, among which each bar of the given framework has at least one corresponding bar, and in each of which the given loading would cause the same kinds of stress in the bars as in the corresponding bars of the given framework. Then, providing the primaries corresponding to the indeterminate frameworks of one superfluous bar are among our series, all the same consequences as before follow, and the given framework cannot be more efficient than the most efficient among these elements. But among these elements the statically determined primary frameworks are always more efficient than the derived frameworks with one superfluous bar; therefore the given framework cannot be more efficient than one of its statically determined elements. (Q. E. D.)

Our first preceding proof was given under one limitation (merely for the sake of simplicity), which was quite unnecessary. We supposed the sections to vary simply as the stress, but it suffices that under the rule of design (post formula) actually employed, the rate of increase in supporting power of a member does not at any time diminish as the area of section is increased. Such is the case for the usual formulæ (for all of the Gordon-Rankine type), so our conclusion holds for designs under the usual rules of dimensioning of members. In such designs the relation between area and stress for each bar having been put in the form $S_i = \phi_i (A_i)$ we would find for the change in area corresponding to a small change in stress $dA_i = \frac{S'_i dS_{m-k+1}}{\phi'_i (A_i)}$ whence the change in volume would be expressed by

$$dV = dS_{m-k+1} \sum_{1}^{m-k+1} \frac{l_i S'_i}{\phi'_i (A_i)}$$

$\phi'_i (A_i)$ replacing σ_i of the simple formula. In order that this expression which now contains the variables $\phi'_i (A_i)$ should not change sign

during variation, at least until some bar stress reduces to zero, it suffices that $\phi'_i(A_i)$ should in no case diminish numerically as A_i increases, the analytic expression of our preceding statement. This holds, as mentioned, for formulæ of the Gordon-Rankine type, and also for the even simpler forms $S = \sigma A - C$ and $S = \sigma A - K\sqrt{A} - C$, which the author believes may find sufficient recommendation for introduction into practice as safe and simple substitutes for the unprofitably complicated, if more correct formulæ.

Illustration of Economic Effect of Variation of Proportions of Statically Indeterminate Frameworks.

To render more clear the significance of the preceding analysis and its application, consider the following simple illustration in which for simplicity we neglect post formulæ and employ the simple rule, stress = constant working stress × section area. Consider the design previously discussed whose stresses are shown in Figure 8, and whose

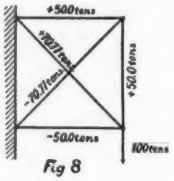

Fig 8

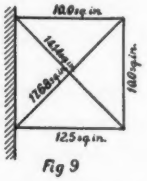

Fig 9

section areas are shown in Figure 9 (both reproduced here). Through the employment of primary stress this design was made economical in character, since all intensities of stress are the maxima allowable. The quantity of material in each of its bars and the total quantity are as

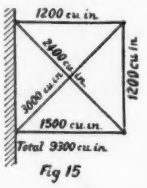

Fig 15

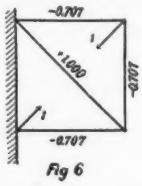

Fig 6

shown in Figure 15. Taking the same diagonal from the lower inner corner to the upper outer corner as the superfluous bar, as before, the stresses S' of our formulæ are those given in Figure 6 (reproduced here), and our formula becomes

$$dV = dS \left\{ \begin{array}{rrrr} \dfrac{(-.707) \times 120 \text{ in.}}{5} & = & & -17.0 \\ + \dfrac{(-.707) \times 120 \text{ in.}}{5} & = & & -17.0 \\ + \dfrac{(+1.000) \times 169.7 \text{ in.}}{5} & = & +34.0 & \\ + \dfrac{(-.707) \times 120 \text{ in.}}{-4} & = & +21.2 & \\ + \dfrac{(+1.000) \times 169.7 \text{ in.}}{-4} & = & & -42.4 \\ & & +55.2 & -76.4 \\ & & & +55.2 \\ & & & -21.2 \end{array} \right\} =$$

$dS \times (-21.2)$ that is, dV is negative when dS is positive. So by increasing the tension (that is, diminishing the compression) in our superfluous diagonal we decrease the amount of material necessary at the rate of 21.2 cubic inches for every ton reduction in our superfluous diagonal, and this rate of saving we may maintain until the stress in some bar reduces to zero. But this happens simultaneously to the superfluous diagonal, the vertical and the upper horizontal, the stress

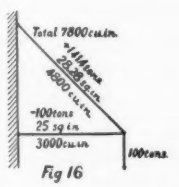

Fig 16

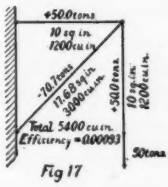

Fig 17

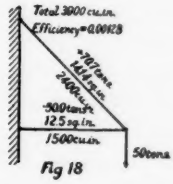

Fig 18

in the superfluous diagonal having diminished 70.7 tons with the consequent saving of $70.7 \times 21.2 = 1{,}500$ cubic inches in material, and our frame having reduced to the simple statically determined frame (triangle) shown in Figure 16. As this frame requires but 7,800 cubic inches the original frame required $\frac{9{,}300 - 7{,}800}{7{,}800}$, or nearly 20 per cent. more material. By Professor Swain's method we should have found the elementary frames (Figures 17 and 18), which together would be equivalent to the given framework (Figures 8 and 9). The efficiency for Figure 17 is .00093, for Figure 18 is .00128, while for the given framework it was $\frac{100}{9{,}300} = .00107$ or intermediate. Clearly, Figure 18 is a more economical form.

The amount of saving shown in this illustration need not be insisted on. With frameworks of other proportions or figure, differences as high as 50 per cent. have been found, while in other instances they were but a few per cent. Employing post formulæ in design the saving comes out slightly greater. The main point to note is that the indeterminate forms, that is, those with superfluous bars, do not employ less material than some included statically determined design of like stresses in kind and intensity.

We may make a very interesting extension of our result. Suppose the bars to be of different materials, so that not the *quantities* but the *values* of change in material employed, are of interest. Let a_i be the value per cubic unit of the material of the bar i, then $\frac{a_i l_i S'_i}{\sigma_i} dS_{m-k+1}$ would be the increased *value* of material employed in the bar i as a result of the increase dS_{m-k+1} in tension in the bar $m - k + 1$, and for the whole framework

$$dC = dS_{m-k+1} \sum_{1}^{i} \frac{a_i l_i S'_i}{\sigma_i}$$

would be the increase in cost of material employed. If a_i be taken constant, this would result as before in a statically determined form being cheaper, with these same materials in its corresponding bars. This applies particularly to construction in different materials, such as wire cable and cast metal, or different substances, as iron and wood. The result is very general and conclusive in negating the claim of a superior economy for construction with superfluous bars, at least for the support of one definite loading.

We might, theoretically, consider further the modifications of cost of abutments, accompanying the changes in the reaction forces during our variations, but practically this would hardly be feasible. However, if, in the process of reducing from an indeterminate to a determinate form, we find it possible to reduce the number of reaction limitations, we may be sure that the cost will only be favorably affected thereby.

One feature of our result should be particularly noted, that in *each* case we can find a more economic statically determined framework among those whose figures are included in the figure of the given indeterminate framework, and whose stresses in its corresponding bars are of the same intensities and *kinds*. Therefore, nothing need be

altered in the manner or material of construction of such a framework which could employ the same materials in its bars as in the corresponding bars of the given framework. However, if we were seeking *the* most economic framework whose figure was included in that of the given framework, and in which we could employ, as we found most advantageous, any materials employed in the given framework, then these limitations would disappear. And it follows from our preceding result that we would only need to seek among the statically determined forms for that furnishing the most economic design.

Thus far we have treated only the case of a single definite loading, and shown that for that the claim of superior economy of material on the part of statically indeterminate construction is untenable. There remain, however, the cases so common in practice, of frameworks which are called on to support a number of different loadings.

Before considering these further, there are certain facts that it will be well to note. Frequently, perhaps usually in practice, some one loading or combination of loadings determines the dimensions of almost all the principal members of the framework, the members not thus determined affecting the total quantity of material employed but slightly. This is particularly the case for large structures in which dead weight plays a leading part, and where the total quantity of material employed is so large that economy in this direction is particularly desirable. In such cases one loading virtually rules the economic design, and our foregoing results hold approximately true. *The consideration of the effects of changes in loading in these cases*, although of great importance as far as strength of the structure goes, *is of wholly secondary significance from our economic point of view.*

It may happen, however, that loadings of comparable importance but widely different character are to be supported. For such cases it no longer necessarily holds that a *single* statically determined framework can be found whose figure is included in that of a given framework, and which will more economically support these several loadings than the given framework. We must here employ several of the statically determined forms in combination.

For example, a weight W could most advantageously be supported by a vertical post, thus ; a horizontal pull P, by a triangular truss, thus ; but the weight being much larger than the pull, a statically indeterminate frame of the form would support either of

these loadings (not both united) more economically than any of the statically determined forms obtainable by eliminating a bar, as . But here *two* statically determined forms taken together will even more economically perform the *two* functions. We have only to design a triangle sufficiently strong to bear the horizontal pull. For the vertical load we calculate how much this triangle will carry, and then design a free vertical post, merely for the balance. In this way there is no interference such as occurs in the indeterminate form, where the diagonals are not permitted by the post to carry their full share of the vertical loading. All parts contribute with their full efficiency.

For other cases we would find similar solutions. For multiple loadings, multiple statically determined forms; such is the dictate of maximum economy. *Statical indetermination in a structure is always to be regarded as self-interference with efficiency.*

Hitherto we have gone on the supposition that it is fair to compare statically determined and indeterminate frameworks, on the basis of the same working stresses. But this is by no means the case. The uncertainty that the conditions of calculation will be fulfilled in construction (we have noted the exactitude as to the lengths of the bars, and the effects of curvature under stress on this), the uncertainty that they will remain as designed (permanent deformation, settlement of abutments, etc.), uncertainties which do not affect statically determined designs, certainly demand much larger factors of safety in the design of statically indeterminate than in determinate construction. Add to this the frequently enormous stresses introduced by temperature changes in indeterminate designs, from which the determinate designs again are free, and all doubts as to with which economy as well as most other virtues lie, will disappear.

We may sum up with the conclusion, that in all cases the best, the most economic results will be obtained through the use of statically determined construction. Let us hope that this may soon be universally recognized, that no more avoidable indeterminate structures such as two-hinged arches and indeterminate suspension bridges will be constructed, and that statically determined construction will become the engineer's ideal.

ONSET, MASSACHUSETTS.
July, 1896.

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